Self-stabilizing and Self-optimizing

Distributed Data Structures

Stefan Schmid (TU Berlin & T-Labs)

Collaborators:
Riko Jacob, Andrea Richa,
Christian Scheideler, Hanjo Täubig, …

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Synchronous model alert!

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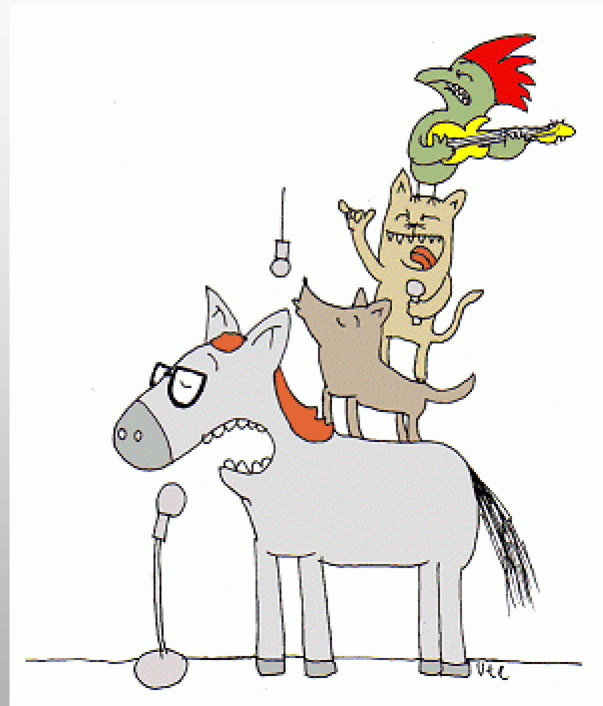
Bremen Stadtmusikanten On A Windy Evening



Bremen Stadtmusikanten On A Windy Evening

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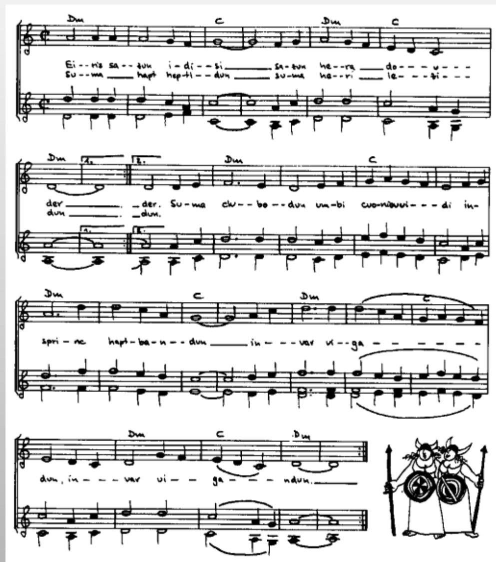

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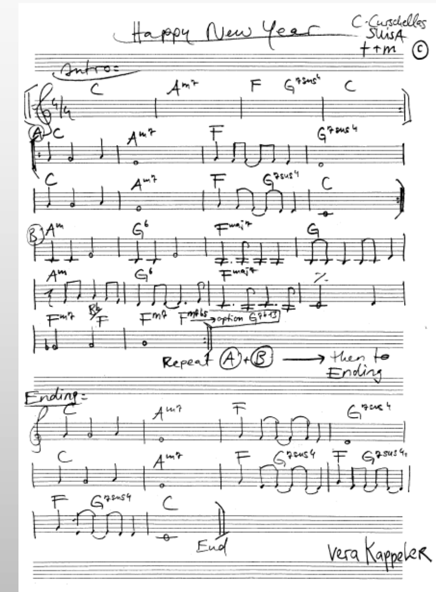
Page 1



Requirements:

- Play “happy birthday” again and again
- Wind changes pages without players knowing!
- When wind stops, harmonize eventually!

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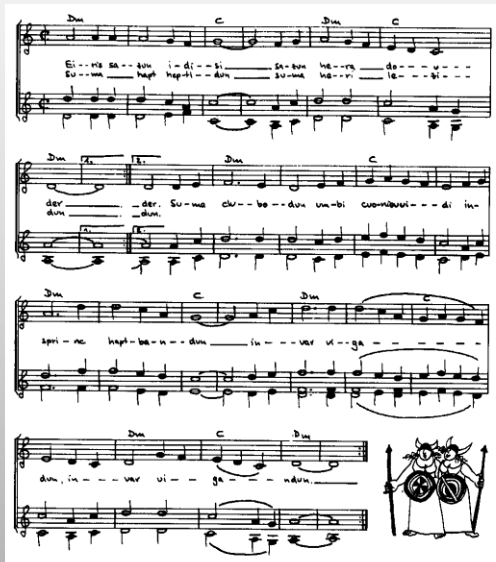

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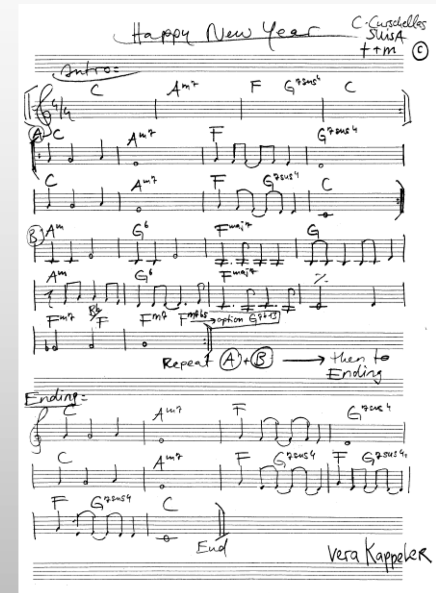
Page 1



How to achieve?

Idea 1: If out of sync, just change to the page of a nearby player!

But what if the neighbor does the same? Do not know who was right! May never converge…

Idea 2: Go to start when asynchrony detected!

But players further away detect it later and restart later! May never converge…

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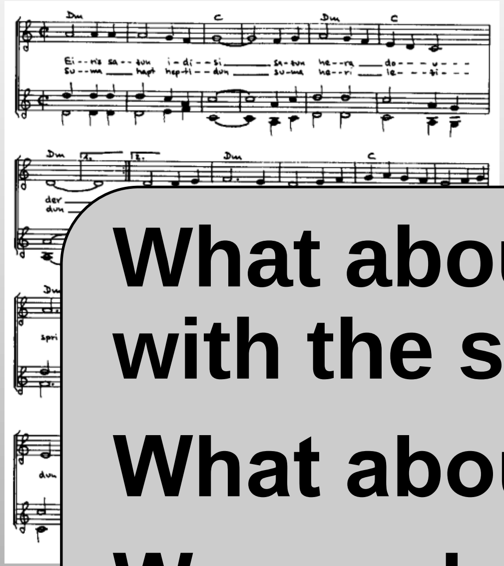

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Page 1

What about synchronizing to the neighbor with the smallest page number?

What about…?

We need a "self-stabilizing algorithm"!

Idea 1: If out of sync, just change to the page of a nearby player!

But what if the neighbor does the same? Do not know who was right! May never converge…

Idea 2: Go to start when asynchrony detected!

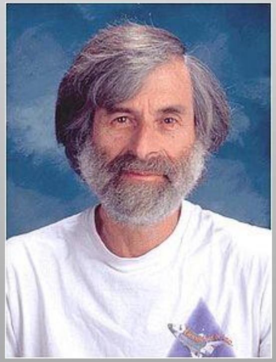
But players further away detect it later and restart later! May never converge…

A Historical Note

Self-stabilizing algorithms pioneered by **Dijkstra** (1973): for example self-stabilizing mutual exclusion.

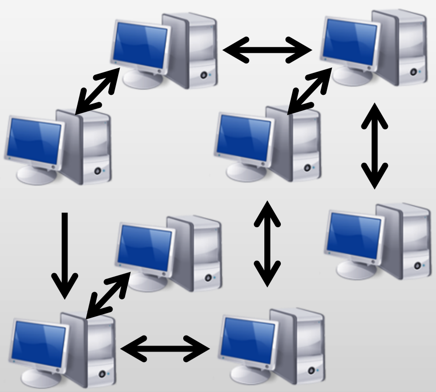
"I regard this as Dijkstra's most brilliant work. Self-stabilization is a very important concept in fault tolerance."

Leslie **Lamport** (PODC 1983)

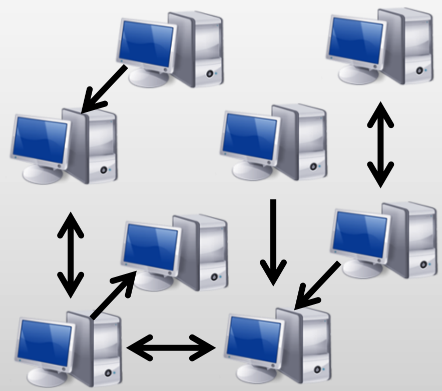


This Talk: Topological Self-Stabilization

From chaos to order: self-stabilizing distributed datastructure (e.g., p2p)

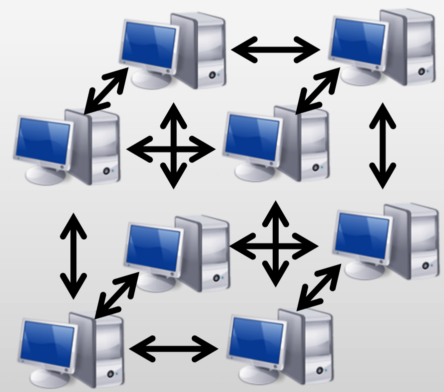


**failures,
adversary,
attack, …**



t_0

adversary stops

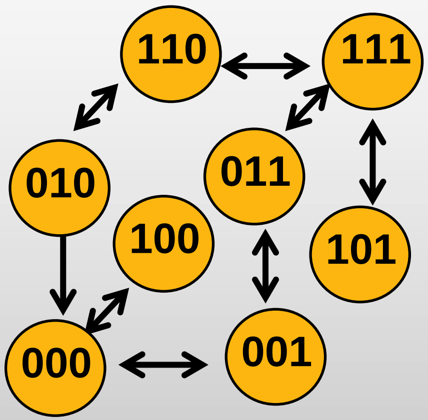


t_0+D

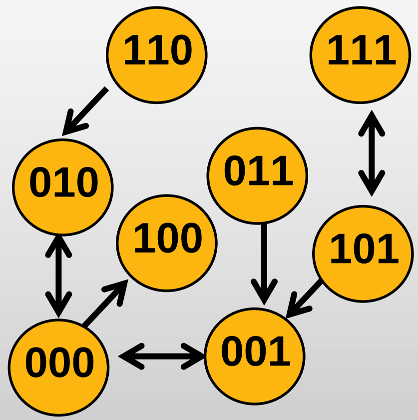
desired structure

Formal View

Example: Hypercube (log+log)

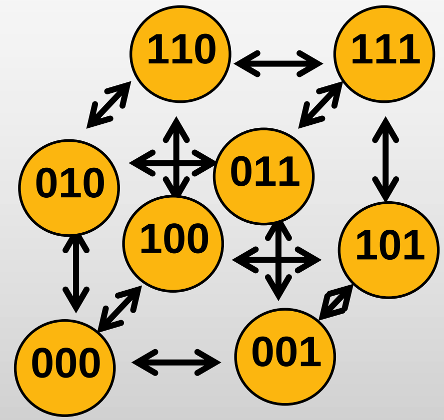


**failures,
adversary,
attack, …**



t_0

weakly connected

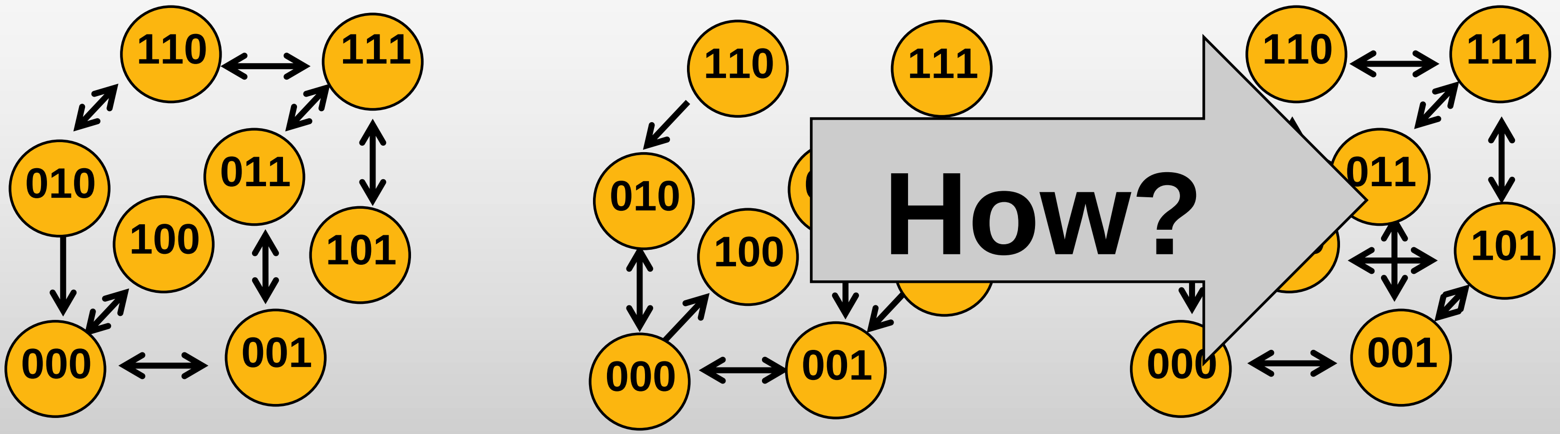


t_0+D

**stabilized
hypercube**

Formal View

Example: Hypercube (log+log)



failures, adversary, attack, …

t_0

weakly connected

t_0+D

stabilized hypercube

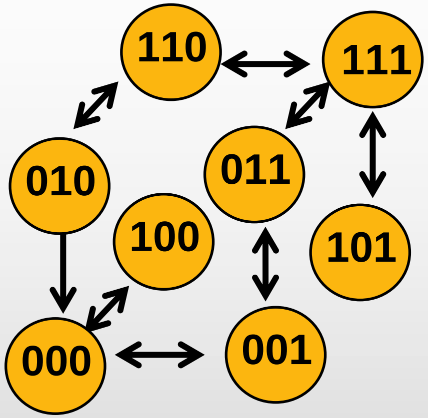
How:

- Distributed: local algorithm
- Fast: minimize D, and stay there!

Model & Terminology

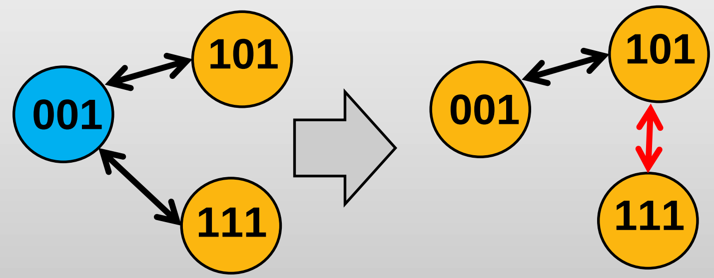
Configuration

- **Constants:** identifiers
- **Variables:** neighborhoods (set of identifiers)
- Union over all nodes



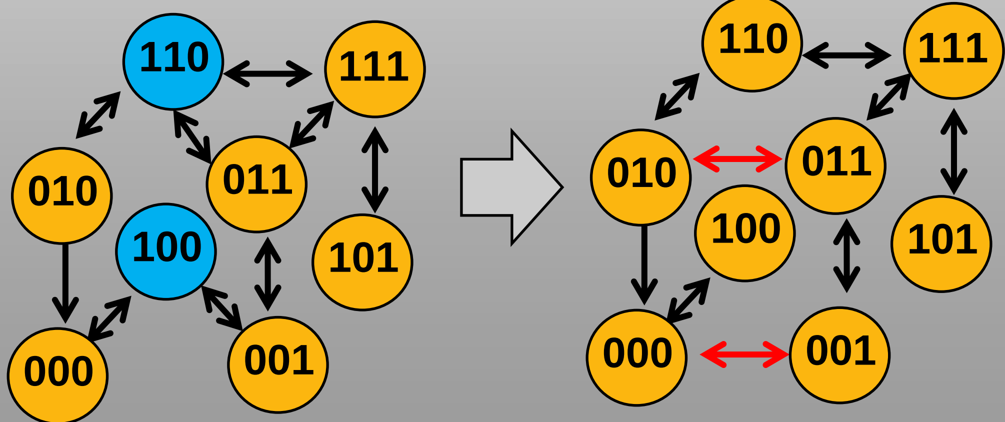
Rules

- **Condition:** on local state
- **Action:** propose new link in neighborhood
- Careful: stay connected!



Execution

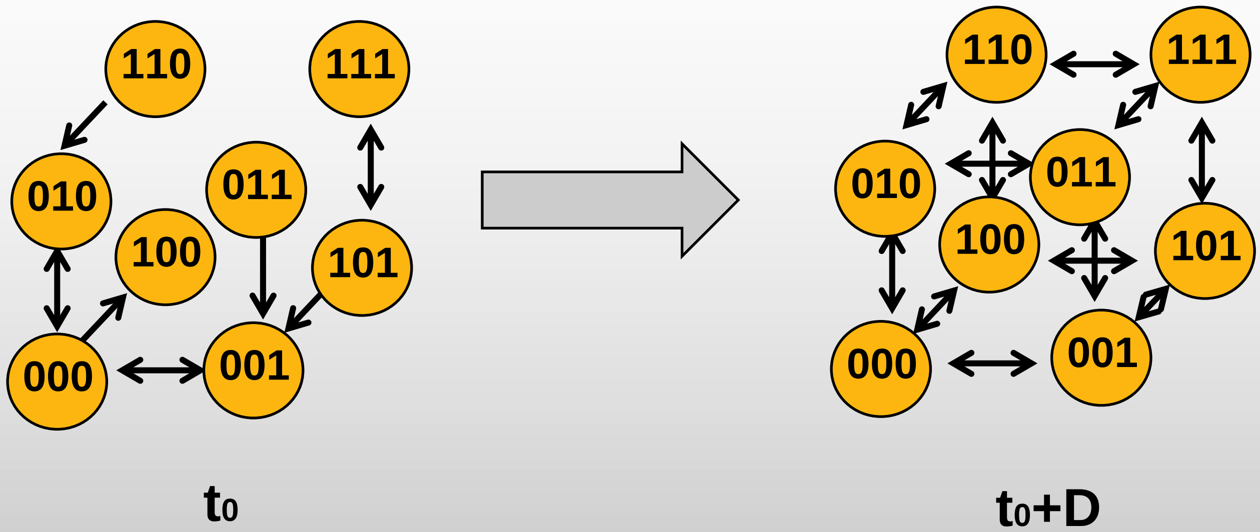
- **Scheduler:** execute enabled actions
- Gives next configuration
- In parallel, or "scalably"



Self-Stabilization

- Convergence: eventually we end up in desired configuration
- Closure: once there, stays there

Performance Metrics



weakly connected **stabilized**

Parallel Time complexity

- Number of parallel rounds until stabilization in the worst case
- Depends on scheduler (scalable: only constant number of enabled actions per node)

Work

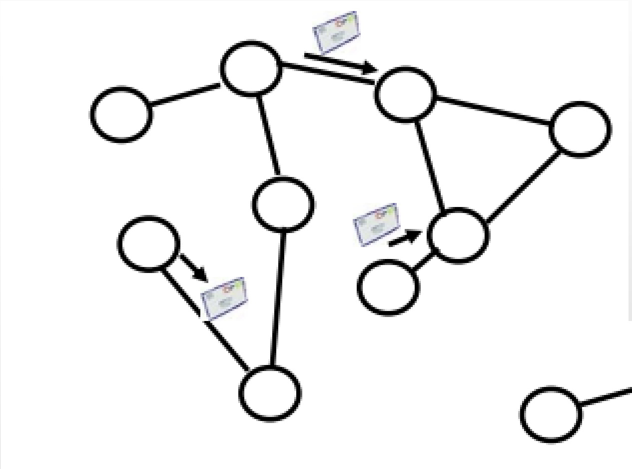
- Number of changed edges

Input-sensitive

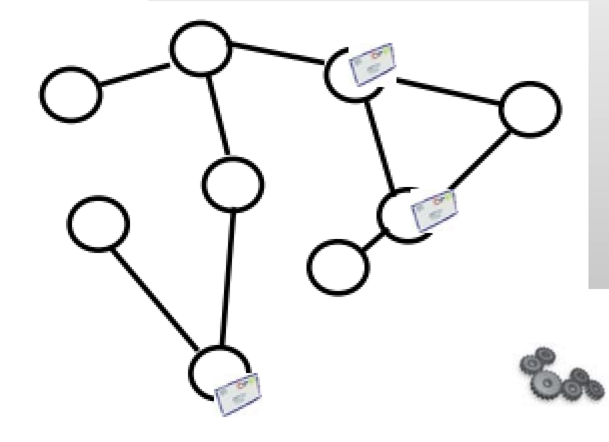
- Local repairs and joins/leaves

Local Algorithms (*LOCAL* Model)

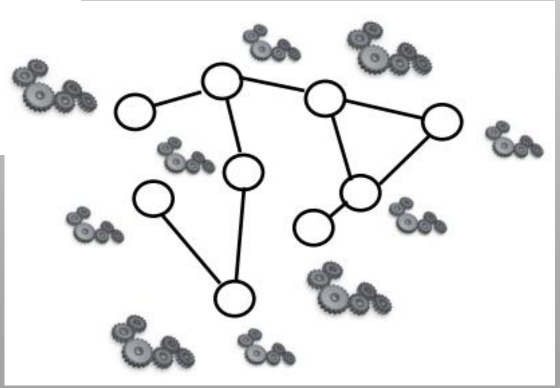
Send...



... receive...



... compute.

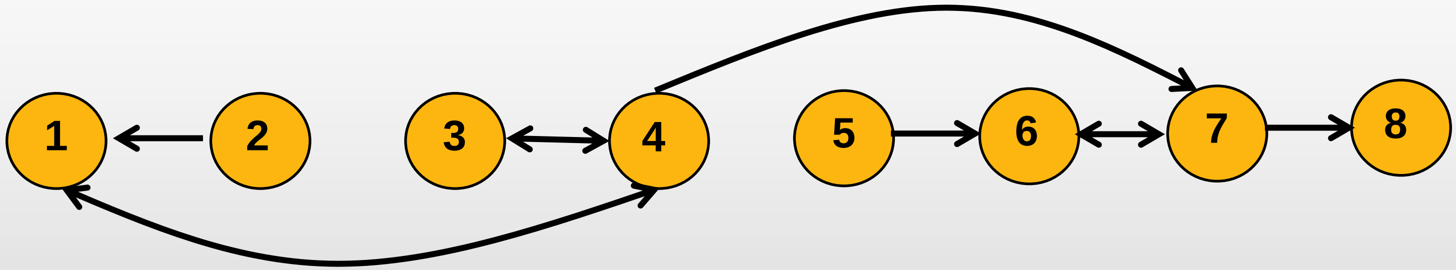


Talk Overview

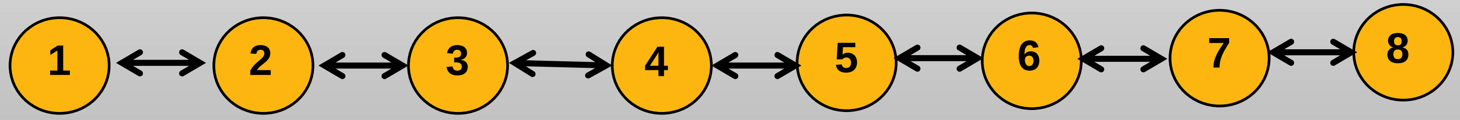
- Primo Piatto: Linearization
- Main Dish: The Skip+ Graph
- Dessert: Delaunay Graphs & Co.
- Digestive: From Self-Stabilization to Self-Optimization

Linearization

Input: Weakly Connected Graph



Output: Sorted Network (wrt IDs!)

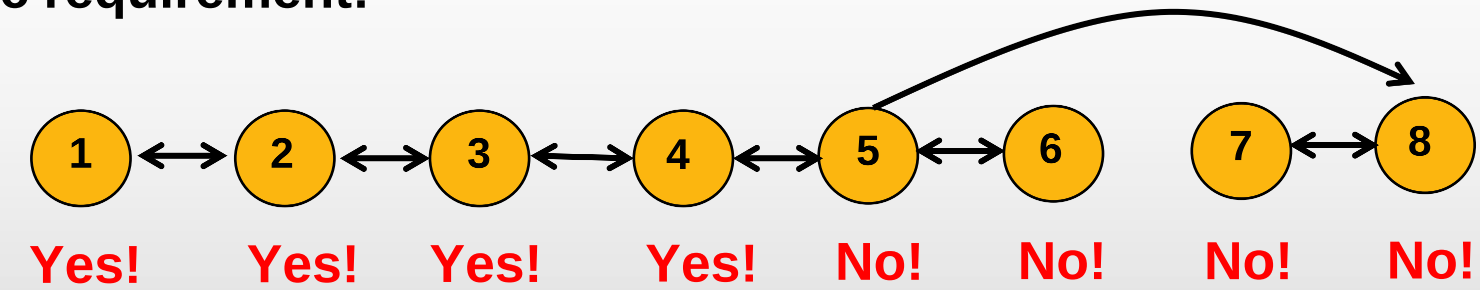


How?

- Local neighborhood changes only
- Preserve connectivity
- Once there, stay there

A First Insight: Local Checkability

Basic requirement:

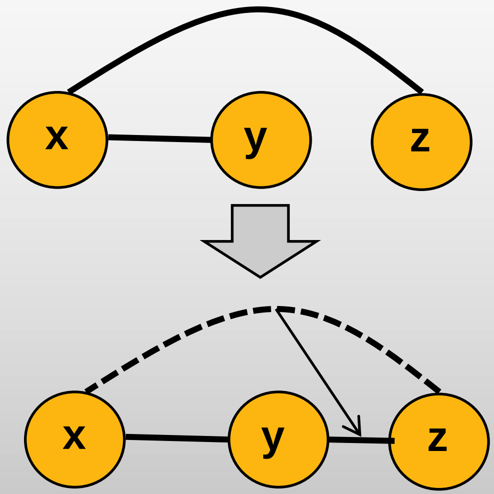


At least one node must observe (and continue changing)!

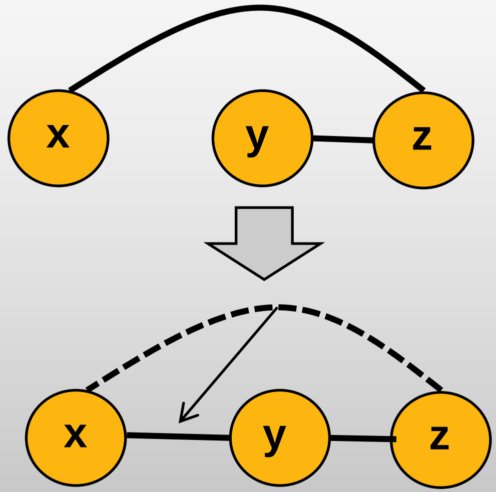
Local checkability:

Most Simple *Undirected* Linearization

Linearize left (x<y<z):



Linearize right (x<y<z):

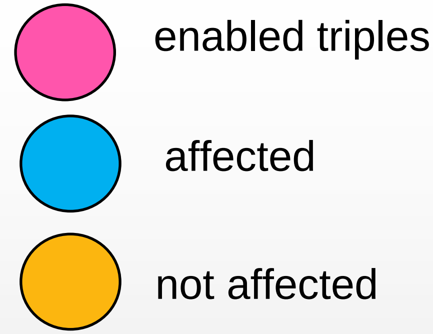


Correctness:

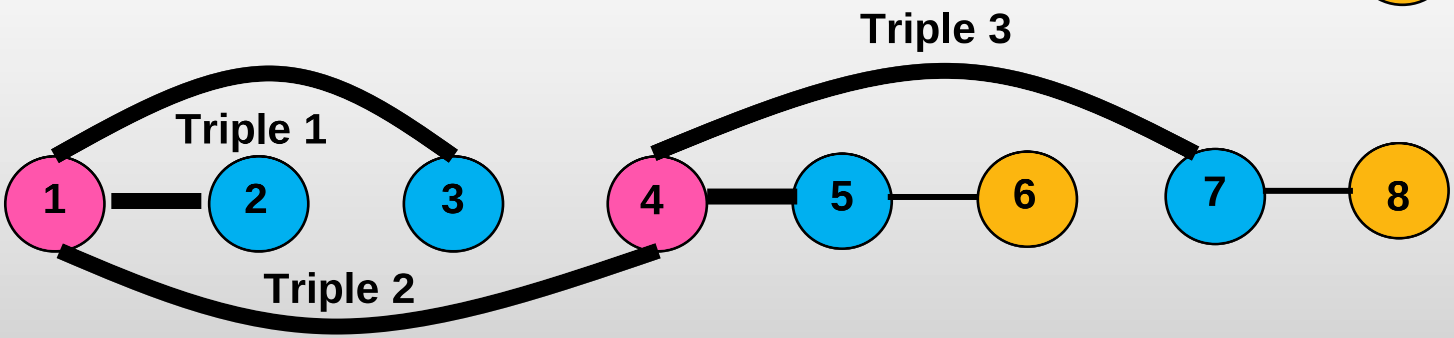
- Connectivity preserved: paths via missing edge still exist
- **Closure:** no changes in linearized setting
- **Convergence:**
  - Triple always exists if not linearized
  - Firing triple reduces potential: $\Phi = \Sigma$ len(e), edges get shorter

Complexity?

Types of Schedulers



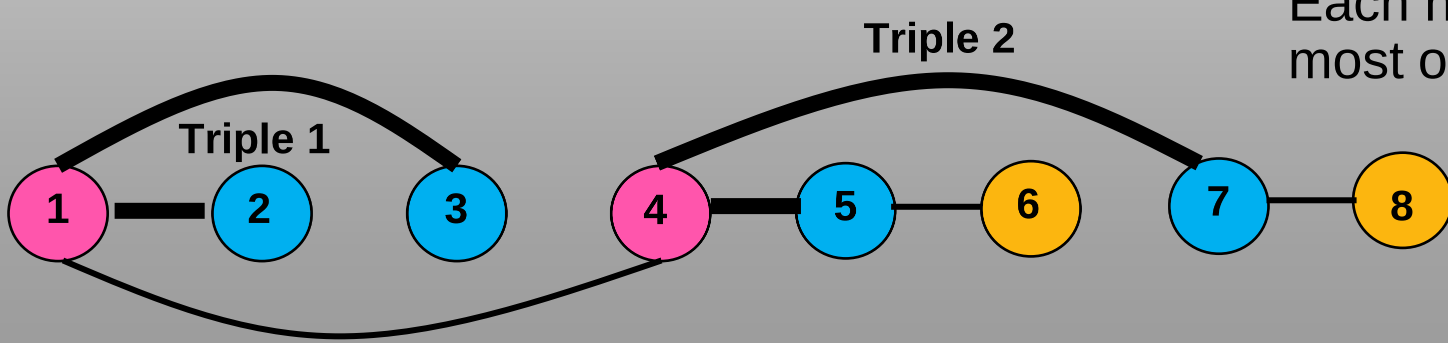
The FULL Scheduler: full set of triples okay



Problem: many changes at single node (e.g., two new edges at node 2, but up to n-1)

Only maximal independent set (MIS Scheduler):

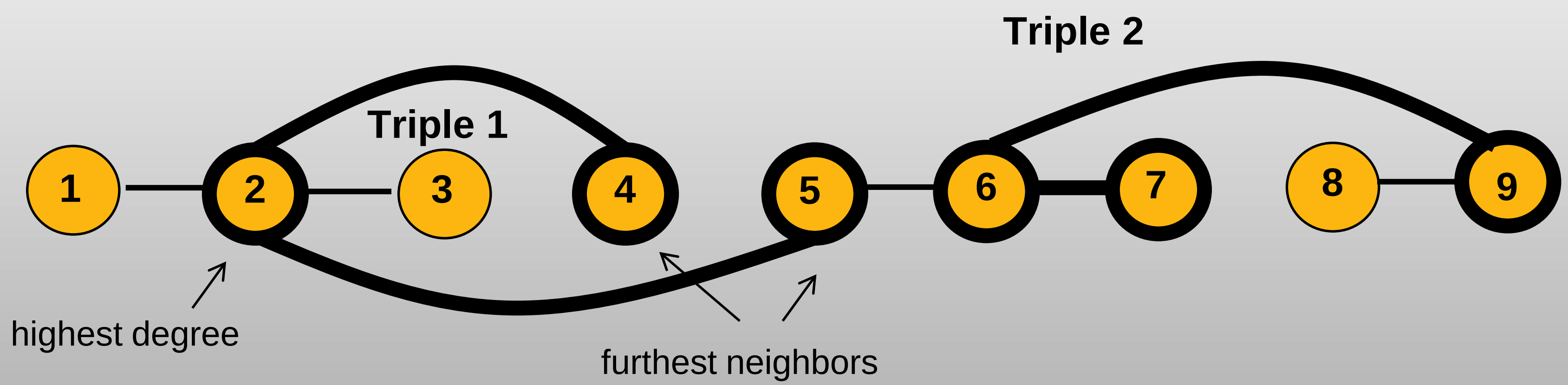
Each node involved in at most one triple!



Concrete MIS Schedulers (Hypothetical!)

Greedy MIS Scheduler

- E.g., select highest (remaining) degree node first ("least linearized guy")
- And for this node, fire triple with most remote neighbors on side with higher degree ("most progress")



Worst / Best Case MIS Scheduler

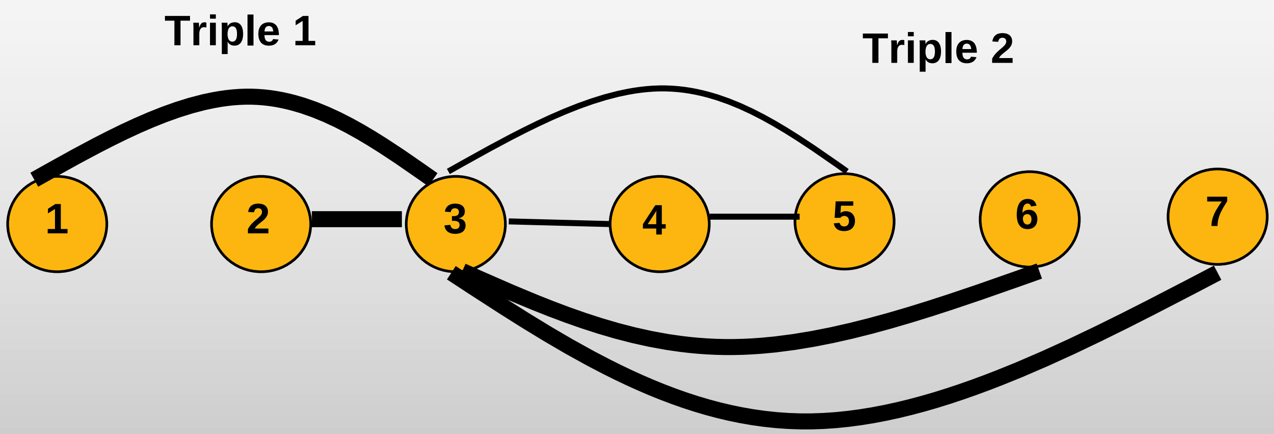
- Worst/best sets of MIS triples such that complexity max/minimized

Random MIS Scheduler

- Random MIS triples

The Algorithm LIN-MAX

The LIN-MAX Algorithm: each node proposes furthest triple on each side



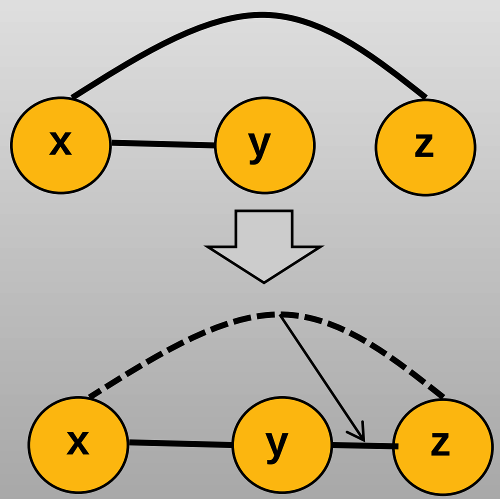
Under a greedy MIS scheduler, LIN-MAX has a time complexity of O(n log n).

Analysis LIN-MAX

Under a greedy MIS scheduler, LIN-MAX has a time complexity of O(n log n).

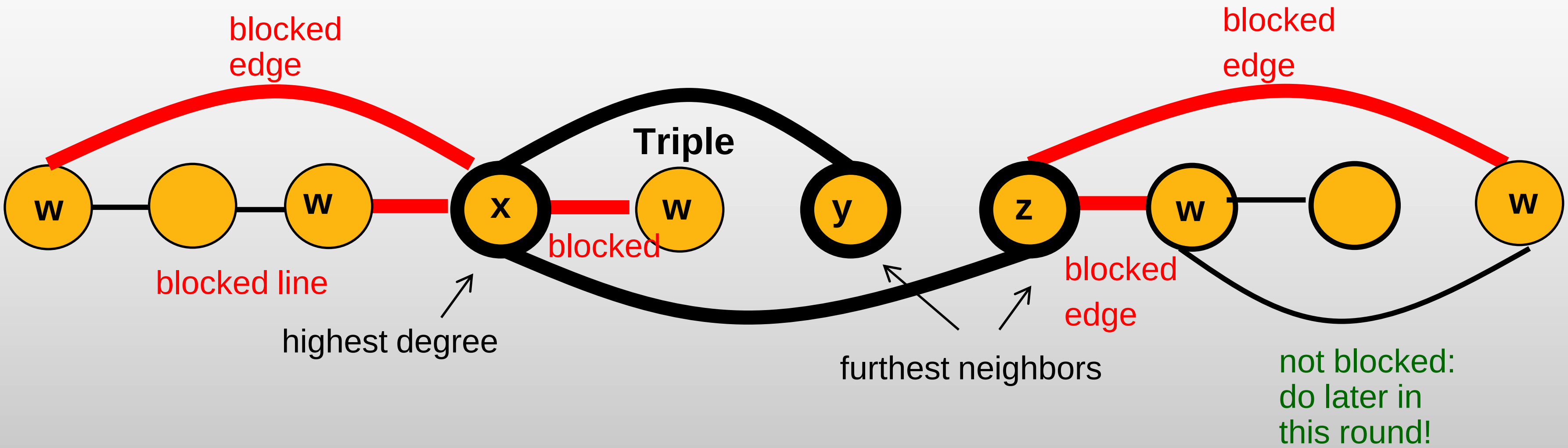
Proof

- Consider potential function **$\Phi = \Sigma$ len(e)**.
- Clearly, initially **$\Phi < O(n^3)$**, each edge at most n long, and when fully linearized, **$\Phi = O(n)$**.
- We show: in each round where triple still exists, potential is multiplied by **factor 1- $\Omega(1/n)$**
- When triple right-linearized by x, Φ reduced by at least **dist(x,z)-dist(y,z)=dist(x,y)**
- Due to greedy degree scheduling, and since LIN-MAX takes furthest neighbors: **dist(x,y) >= deg(x)/2-1 >= deg(x)/4**.
- Due to this triple, how many other triples cannot be fired in this round ("blocked potential"): overall potential at most **O(n)*deg(x)**. So we reduce a 1/n fraction of the total potential. QED.



Blocked Potential

- “Due to the triple (x,y,z), at most **O(n)*deg(x)** remaining potential is blocked.”

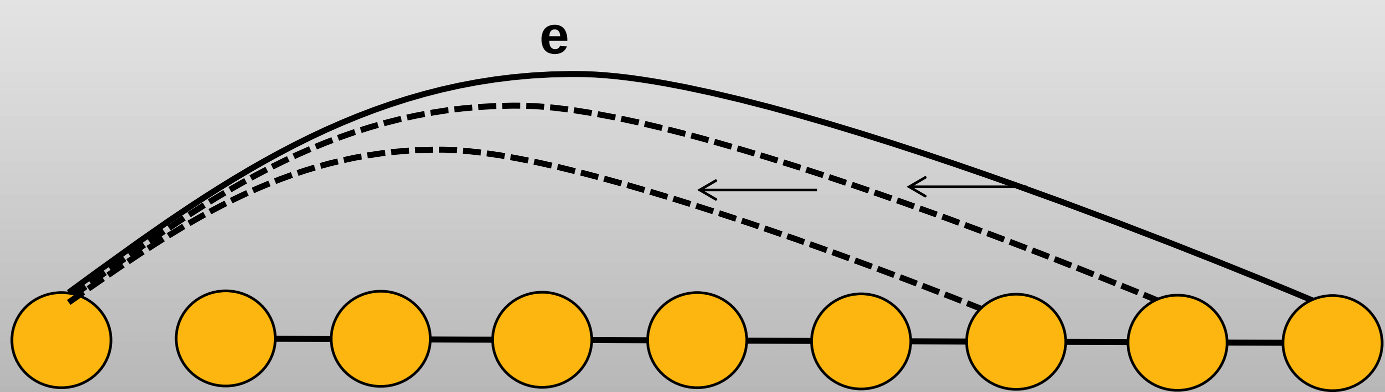


- Look at remaining components and neighbors w of x, y, or z
- Case A: if remaining component is line, cannot linearize further in this step, but line has blocked potential n, plus potential for edge to w (at most n as well)
- Case B: if remaining component still has triples that can fire in this round, account for them later. But lose edge to w (potential n).
- Since **max(deg(y), deg(z)) =< deg(x), max 6 deg(x) neighbors on both sides**
- So we block at most 6*deg(x) edges and components of potential 2n.

A Lower Bound

Even under an *optimal* MIS scheduler, LIN-MAX has a time complexity of Ω(n).

Proof



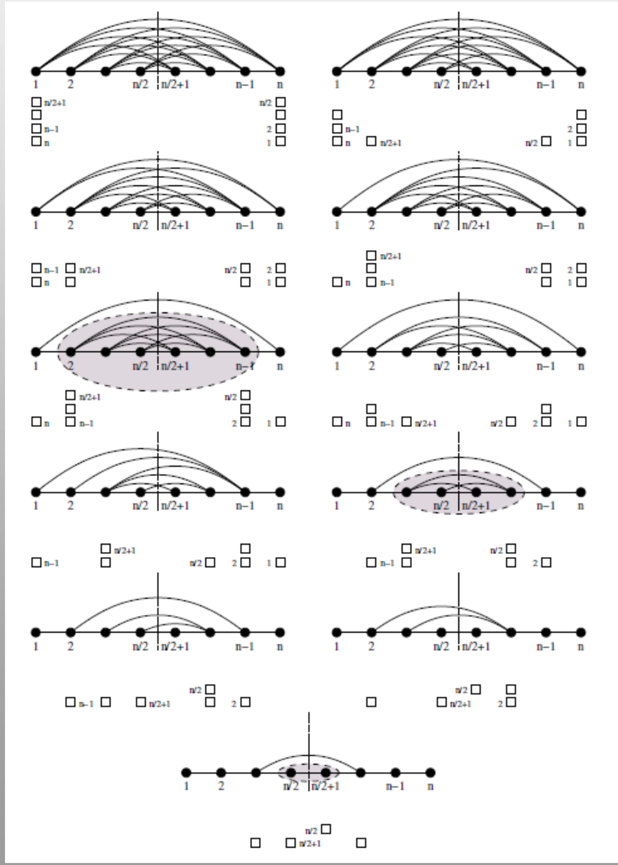
Length of edge e is reduced by one only per round: no parallelism.

QED

A Better Lower Bound

Under worst scheduler, time $\Omega(n^2)$ for LIN-MAX algorithm.

Initially complete bipartite graph.



QED

Talk Overview

- Primo Piatto: Linearization
- Main Dish: The Skip+ Graph
- Desert: Delaunay Graphs & Co.
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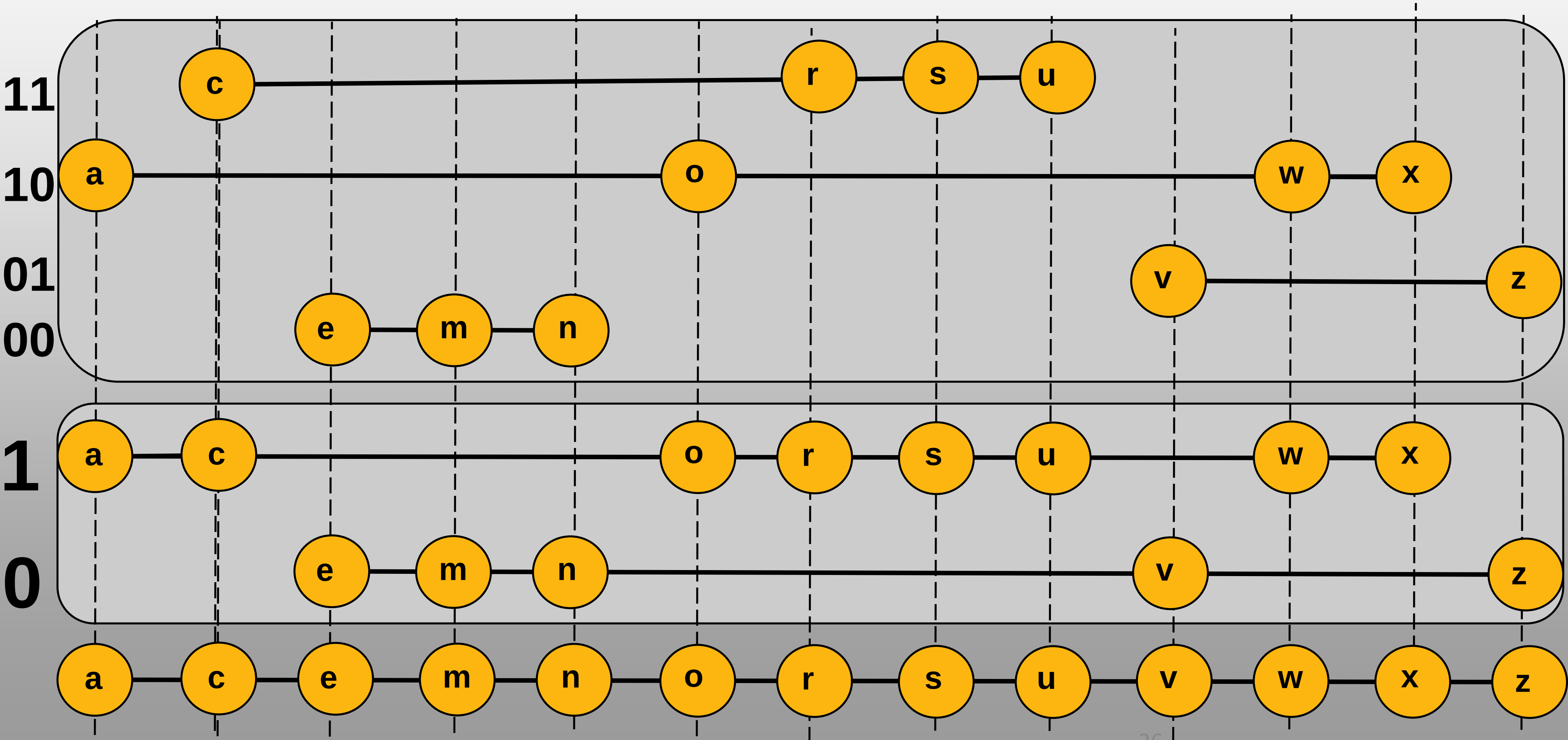
Skip Graphs

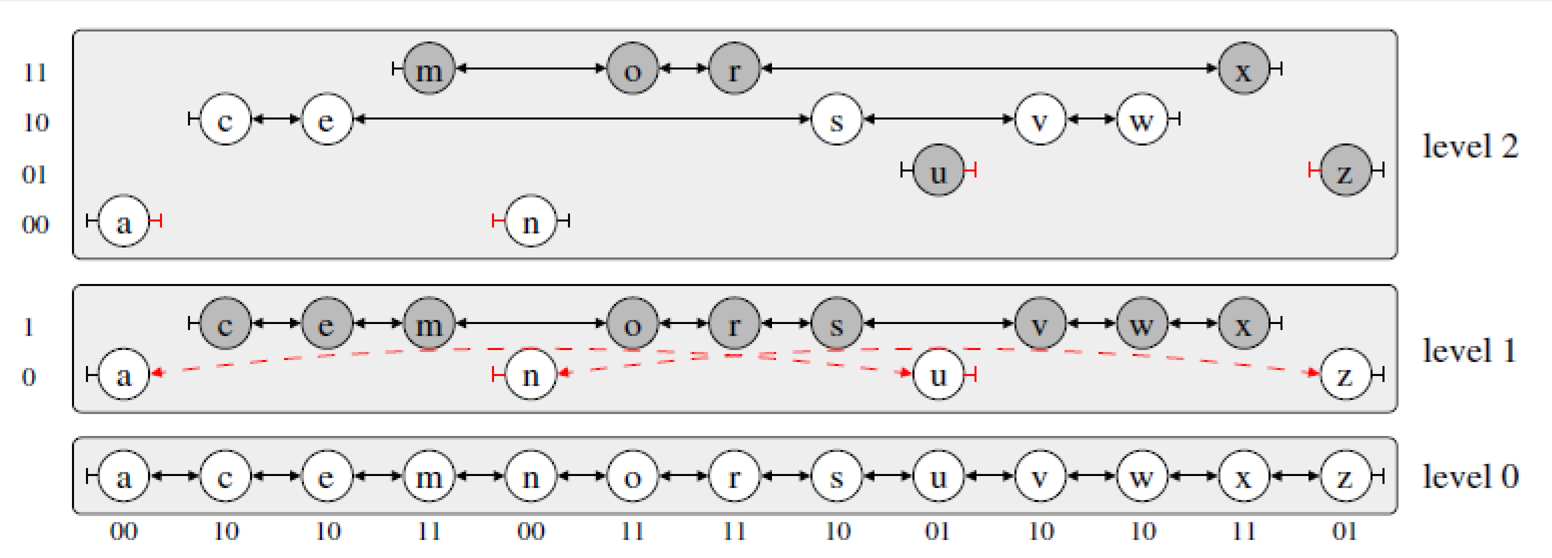
- Attractive distributed data structure: logarithmic height, logarithmic degree
- Distributed variant of skip list…: connect to nearest neighbors on log(n) many levels
- Nodes v have identifier v.id and random string v.rs
- Nodes sorted according to v.id (range search), and organized in layers according to v.rs



Skip+ Motivation: Local Checkability

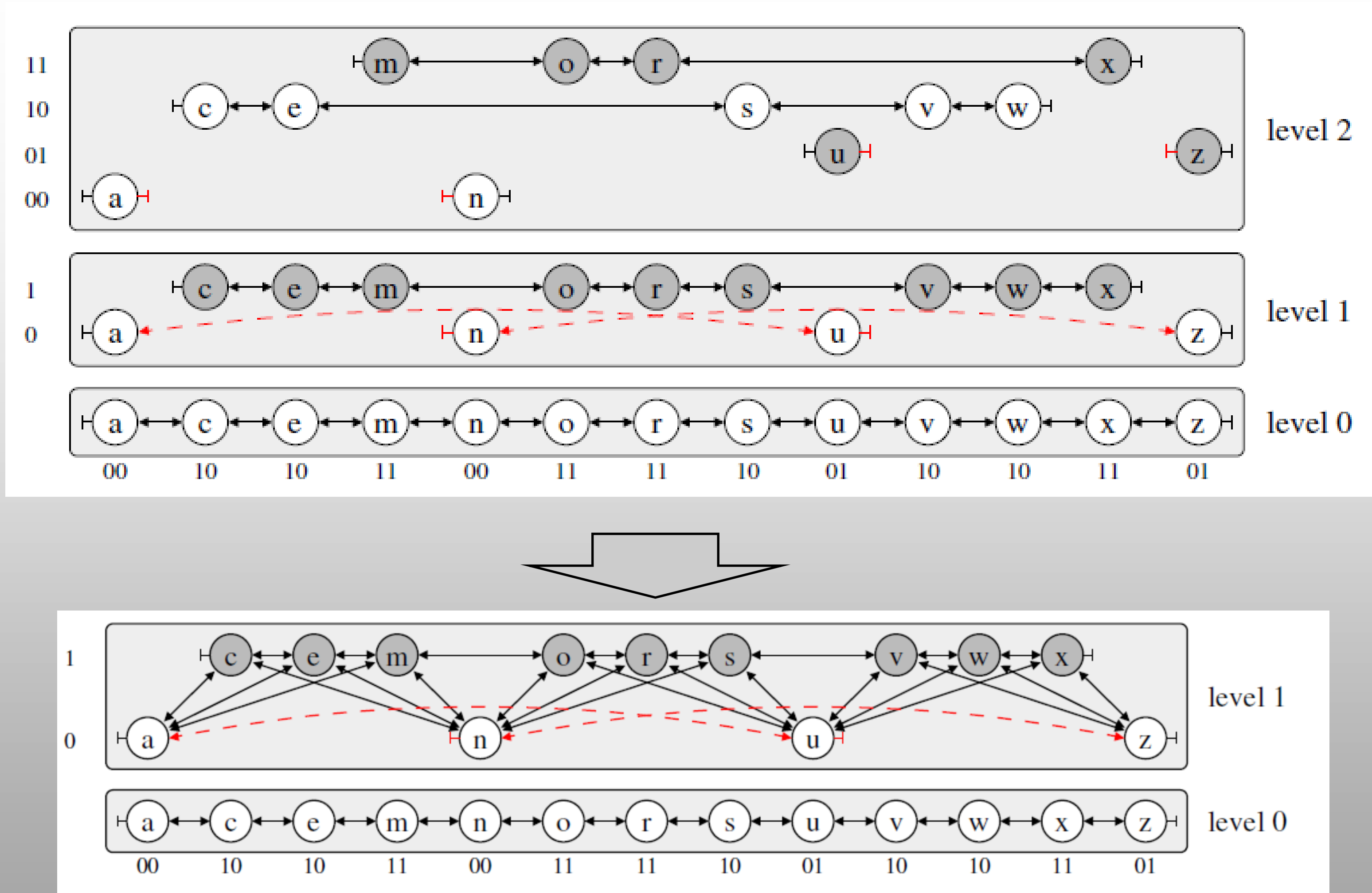
- For fast self-stabilization, we use a different variant of the Skip graph
- Additional edges for (1) local checkability and (2) efficiency

Problem: The following graph looks "locally correct"!



- Such a Skip graph does not work: node a only has two neighbors c (on level 0) and u (on level 1)
- But neither a, c, or u are aware of n: the graph looks correct locally: everyone has its nearest neighbors!

Skip+: Solution By Additional Edges



- Add additional edges to *all* nodes on this level, until nearest neighbor of the prefix
- Node c can now realize that u is not a nearest neighbor of a, and tell it to a!

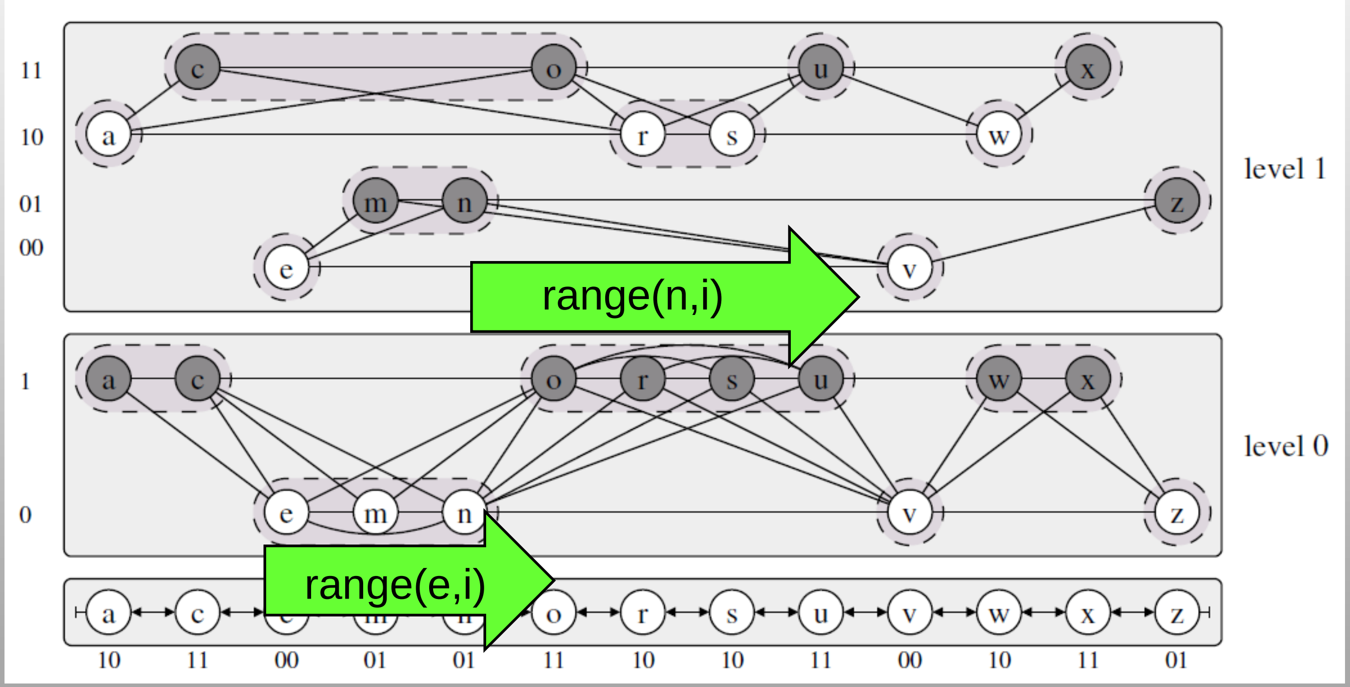
Definition of Skip+

Define predecessors and successors on each level

$$pred^*_i(v,x) = pred(v, \{w \in V \mid pfx_{i+1}(w) = pfx_i(v) \circ x\})$$
$$succ^*_i(v,x) = succ(v, \{w \in V \mid pfx_{i+1}(w) = pfx_i(v) \circ x\})$$

Define ranges in which nodes are interested on a level i: all up to end of opposite color

$$low^*_i(v) = \min\{pred^*_i(v,0).id, pred^*_i(v,1).id\}$$
$$high^*_i(v) = \max\{succ^*_i(v,0).id, succ^*_i(v,1).id\}$$
$$range^*_i(v) = [low^*_i(v), high^*_i(v)]$$



In words: a white node is interested in all nodes until first black node (inclusive); if white node does not have white neighbor yet on that side, it is interested in all black nodes until white again (exclusive).

Properties of Skip+

The diameter and degree of Skip+ is O(log n), w.h.p.

- The height and diameter is not larger than in the original Skip graph
- Interestingly, also the degree does not increase asymptotically
- Probability that there are k neighbors on level i: 2^{-k}
- Union bound over all possible distributions of degrees over levels:

$$\Pr[X = d] \leq \sum_{k_0,\ldots,k_{H-1} \geq 0 : \sum_{i=0}^{H-1} k_i = d} \prod_{i=0}^{H-1} \frac{1}{2^{k_i - 2}} \leq \binom{d+H-1}{H-1} \frac{1}{2^{d-2H}}.$$

If $d = c \cdot (H-1)$, we get

$$\binom{d+H-1}{H-1} \frac{1}{2^{d-2H}} \leq \frac{[(c+1)d]^{H-1}}{2^{c(H-1)-2H}} \leq \frac{[(c+1)d]^{H-1} \cdot 2^{4(H-1)}}{2^{c(H-1)}} \leq \frac{1}{n^{c'}}$$

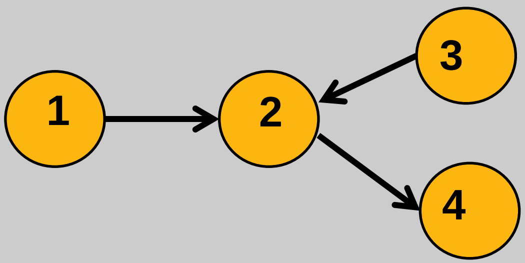
Principles

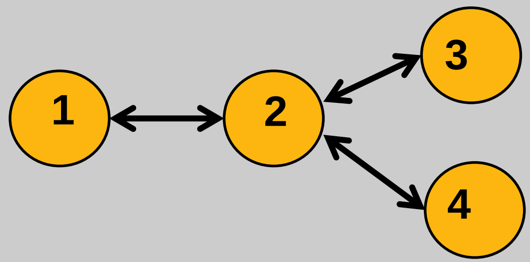
- Never delete any edges! Only forward or merge with existing ones (preserves connectivity)
- Four simple rules: all executed at all times
- No phase changes ("first clique, then…": not self-stabilizing)
- But analysis in phases okay!
- Preprocessing / transition step between rules: make things bi-directed, etc.
- Do not introduce unnecessary edges (degree!)

Distributed Self-Stabilization: Algorithm ALG+

Pre-Processing / Transition

- Receive requests
- Make links bi-directed and send state / neighborhood

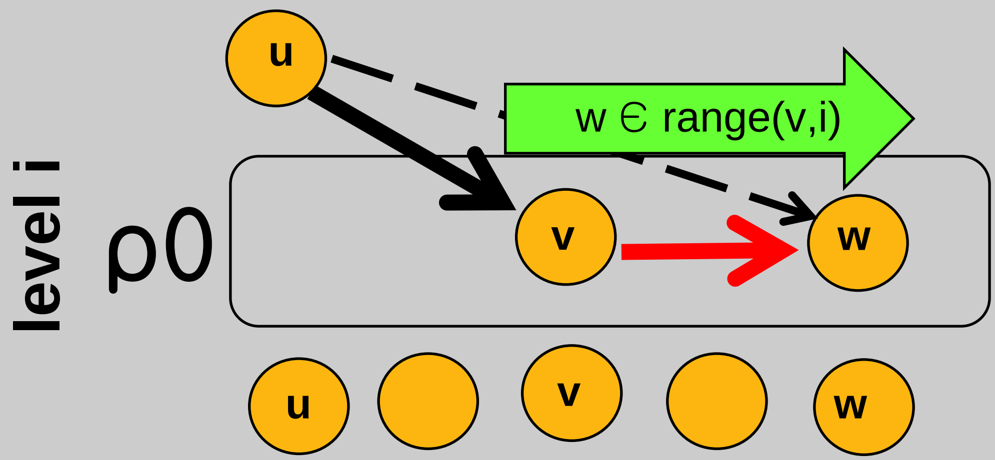




Distributed Self-Stabilization: Algorithm ALG+

Rule 1: Range Reduction

- Distinguish: stable and temporary neighbor
- Stable = out-neighbor in-range on some level i; temporary = not
- Note: in-range at level i implies in-range at level j>i (if prefix still fits: on higher levels less nodes as more prefix bits required)
- For every level i, for any stable v∈N(u) and pfx-i (v)=pfx-i(w) and v interested in w, u requests new stable (v,w), plus if also stable: (w,v)

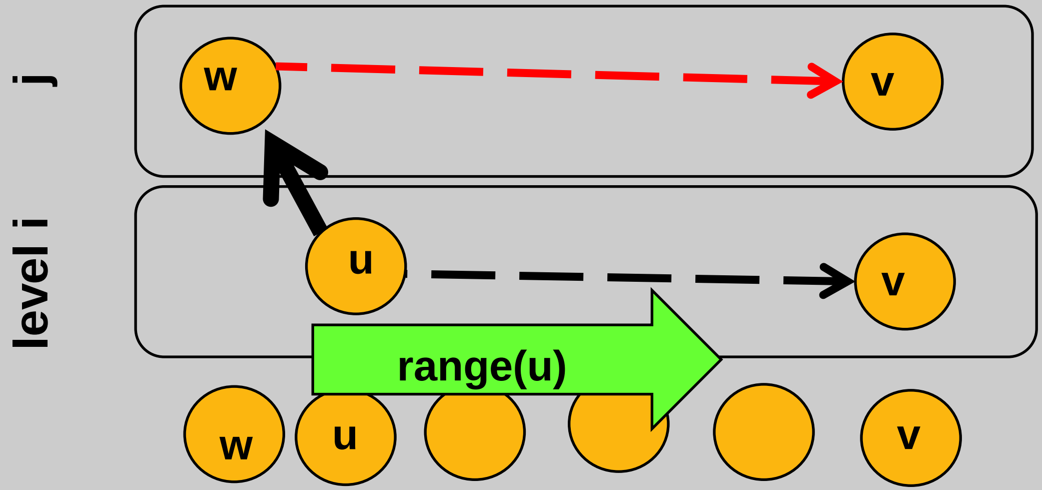


Rule 1 ensures a fast "pointer doubling" until first interesting nodes are found! (Initially: unbounded ranges!)

Distributed Self-Stabilization: Algorithm ALG+

Rule 2: Forward Edges

- Node u forwards non-interesting / temporary edge to (u,v) to the stable neighbor with the largest common prefix with v
- W must exist, otherwise (u,v) would be stable

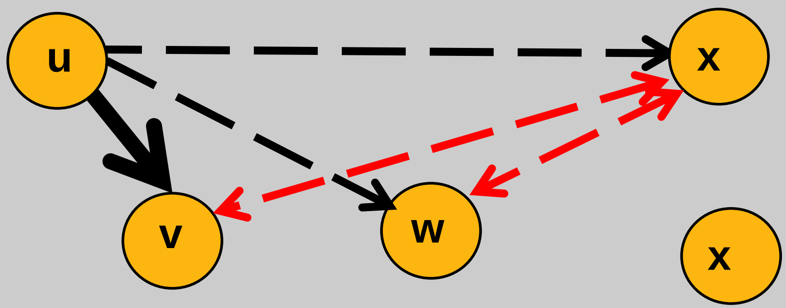


Rule 2 used to quickly propagate edges to nodes where they are more useful (otherwise vanish / merge)

Distributed Self-Stabilization: Algorithm ALG+

Rule 3: Local Closure («intro all»)

- Whenever the stable neighborhood of a node u changes, e.g., even if only the level at which some edge is stable, u introduces edges between all its neighbors

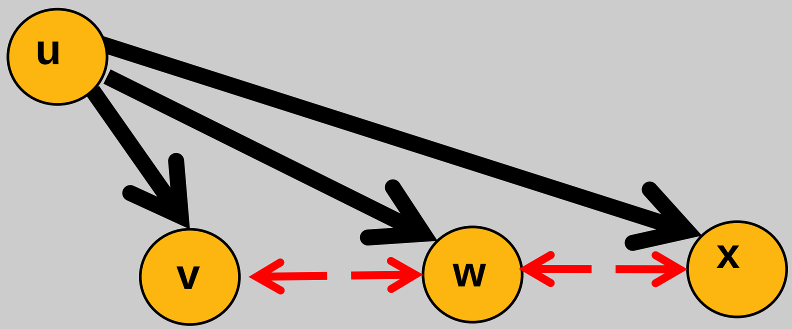


Rule 3 quickly propagates new edges in neighborhood and ensures that already connected components stay connected in future.

Distributed Self-Stabilization: Algorithm ALG+

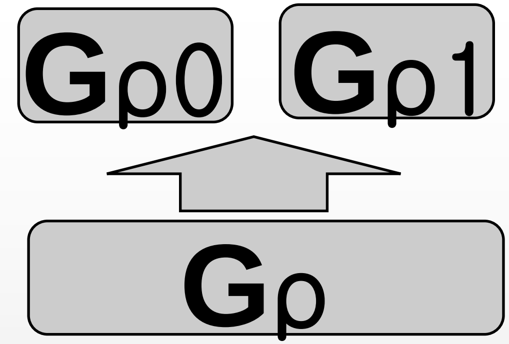
Rule 4: Linearize

- Every node u on every level i: for all stable neighbors, link neighbors according to order of identifier ID (not random string rs!)



Rule 4 sorts nodes according identifiers: gives desired search structure.

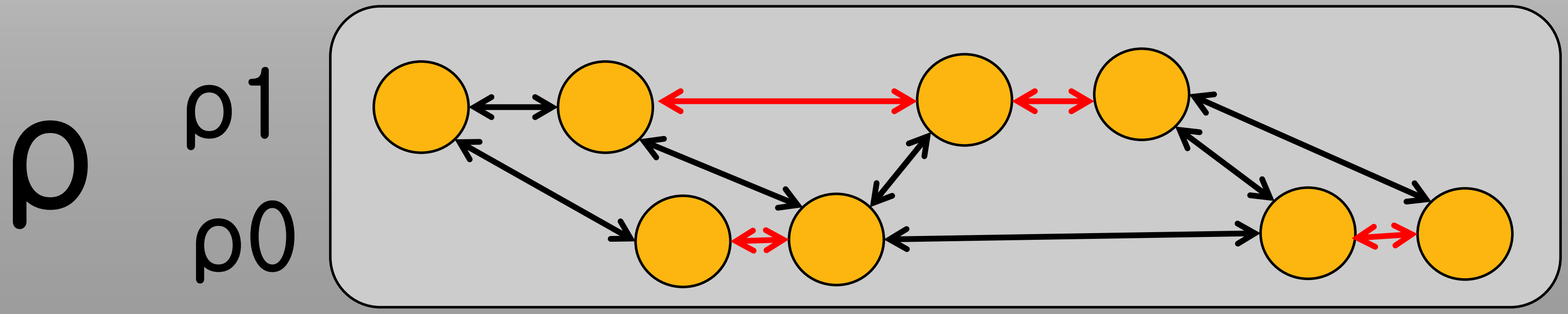
Proof Overview (1)



- Think in “phases”!
- **Bottom-up phase (time $\log^2 n$)**
  - From layer 0 upward, Gρ components arise:

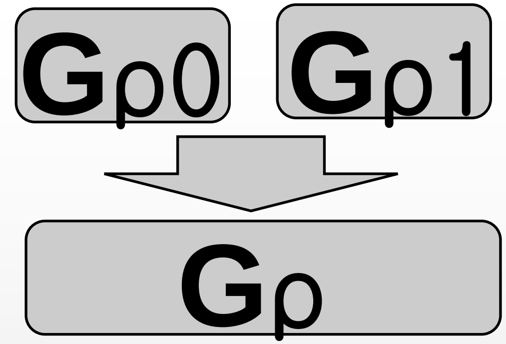
Gρ= (Vρ, Eρ) where Vρ is set of nodes with prefix ρ and Eρ are edges between Vρ nodes.

  - Trivial for empty prefix (connected)
  - Induction: each node with ρ0 finds buddy: node of opposite color ρ1 on same level | ρ |.
  - Given a buddy and connected Vρ, we quickly get connected graphs $G_{\rho 0}$ and $G_{\rho 1}$.

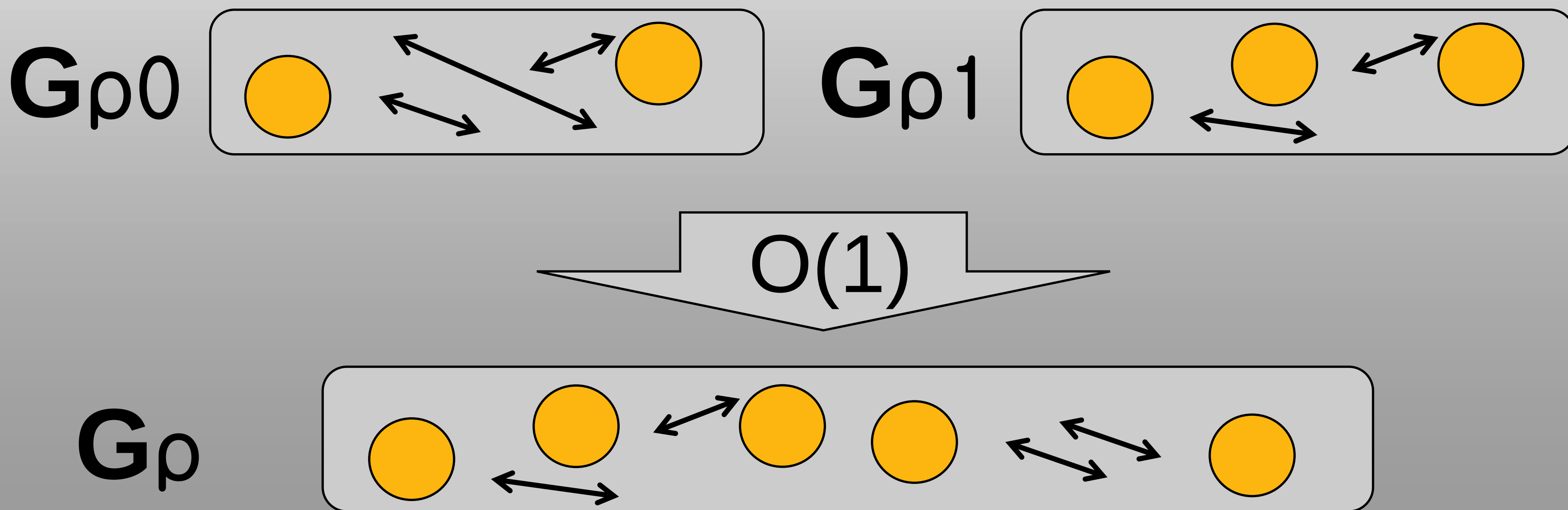


powered by Rule 1 and Rule 3

Proof Overview (1)



- Think in “phases”!
- **Top-down phase (time log n)**
  - From level H downward
  - Level i contains all edges of Gρ (stable “little Skip graph”)
  - Level H trivial: single nodes
  - Then, by Rule 1, two i-finished components zip together to (i-1)-finished component



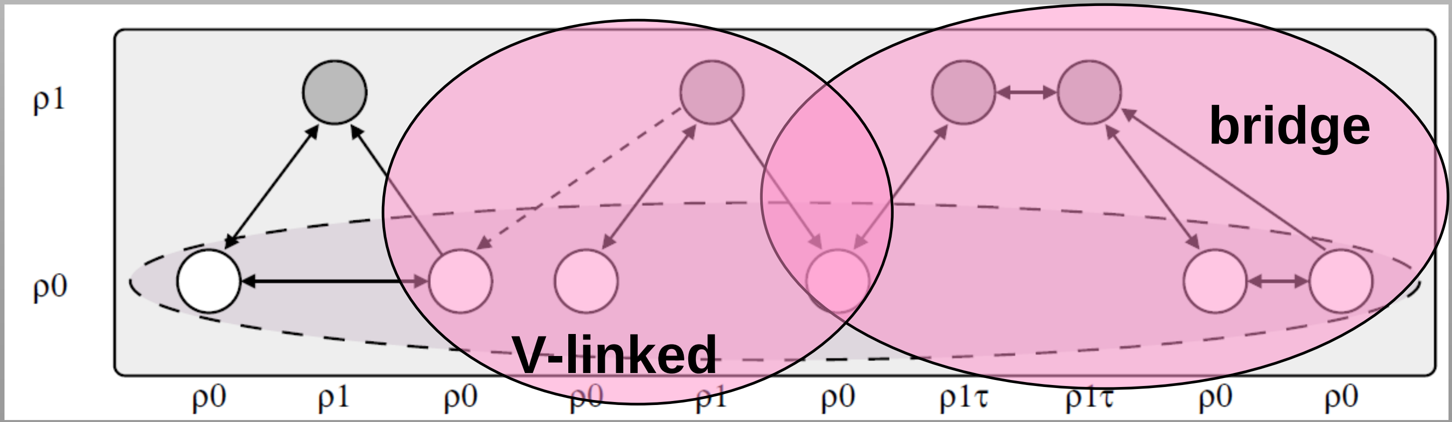
Bottom-Up Phase (1)

- **Lemma: If weakly connected at t0, nodes will have buddy at t0+O(log n) w.h.p.**
  - By Rule 1 (pointer doubling until in-range node!)
- Concept of pre-component / pre-connected:

(σ,k)-pre-component

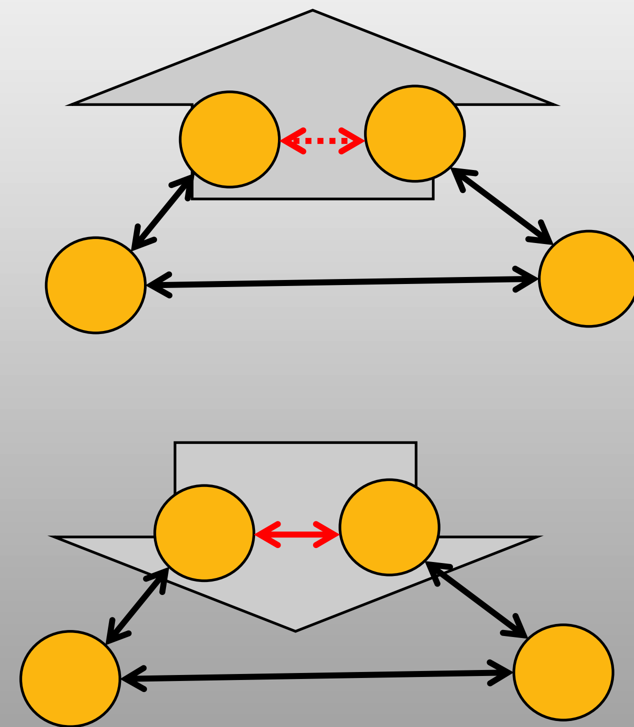
Nodes a, b with prefix σ=ρx but in different Gσ components are (σ,k)-pre-connected if (1) Gρ is weakly connected, (2) every node in Gρ0 and Gρ1 has at least one neighbor in the opposite component, (3) a and b are either directly connected, σ-V-linked, or if there exists a stable (σ,k')-bridge with k'=<k.

Shaded nodes are (ρ0,k)-pre-component:



Bottom-Up Phase (2)

- **Lemma: Once pre-connected, stays pre-connected.**
- **Lemma: If in (σ,k)-pre-component at t, σ-connected at t+4.**
  - Mostly due to Rule 1 and 3
- **Lemma: Evolution of bridges**
  - The level of temporary bridge edge grows quickly: endpoints share larger prefix in each round (Forwarding Rule 2 plus existence of buddy)
  - Then, bridge edge stabilizes, and can serve for forwarding as well.
  - This yields new stable bridges at lower levels.



- **Lemma: Once** Gρ **connected at time t,** Gρ0 **and** Gρ1 **also connected at time t+O(log n).**
- **So summing over all levels: O(log² n).**

Other Features of Skip+

Individual joins/leaves can be handled locally, with polylog work.

Talk Overview

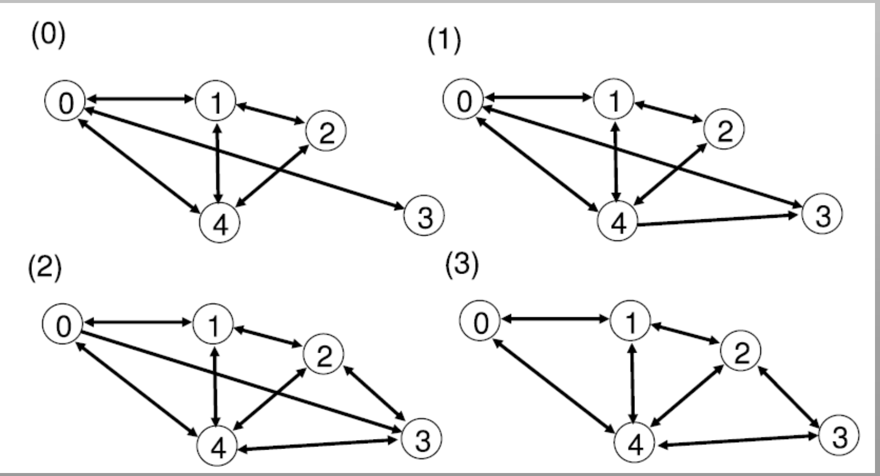
- Primo Piatto: Linearization
- Main Dish: The Skip+ Graph
- Desert: Delaunay Graphs & Co.
- Digestive: from Self-Stabilization to Self-Optimization

Delaunay Graphs

There exists a self-stabilizing algorithm for Delaunay graph with time complexity of $O(n^3)$.

Idea

- More geometric
- Always compute local Delaunay graph of (outgoing) neighbors plus "local hull": stable edges
- Greedily route temporary edges towards node closest to edge destination ("distance compass routing"): maintain connectivity



Talk Overview

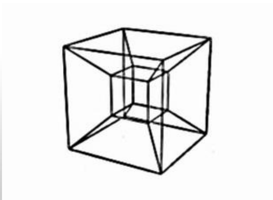
- Primo Piatto: Linearization
- Main Dish: The Skip+ Graph
- Desert: Delaunay Graphs & Co.
- Digestive: from Self-Stabilization to Self-Optimization

From "Optimal" Networks to Self-Adjusting Networks

- Networks become more and more dynamic (e.g., flexible SDN control)
- Vision: go beyond classic "optimal" static networks
- Example: Peer-to-peer

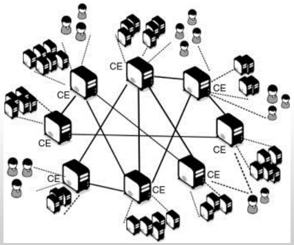
Chord, Pastry, SHELL

- Hypercubic
- Log diameter
- Log degree
- Log routing



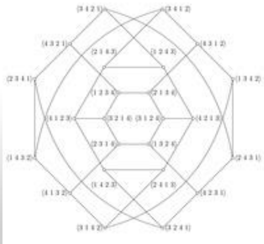
Koorde, ...

- Constant degree
- Log routing



Pancake

- Log/loglog degree and log/loglog routing



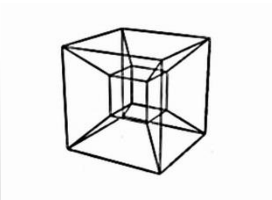
From "Optimal" Networks to Self-Adjusting Networks

- Networks become more and more dynamic (e.g., flexible SDN control)
- Visio
- Exar

What if networks could self-adjust depending on communication pattern?

Chor

- Hyp
- Log d
- Log degree
- Log routing



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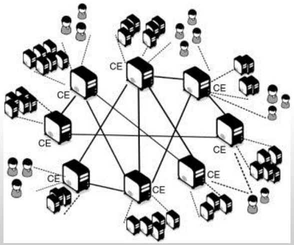
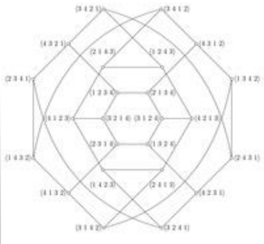
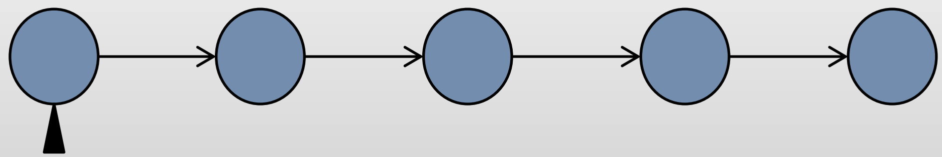




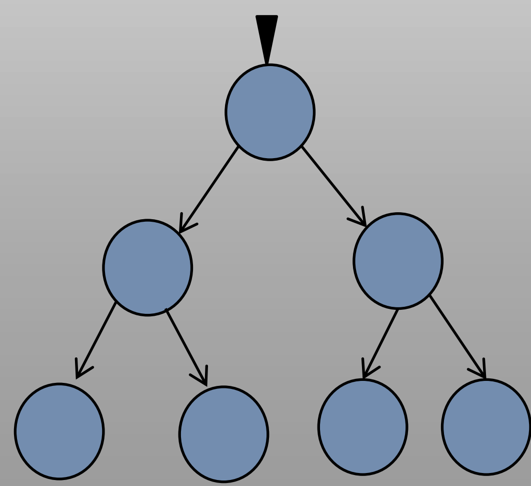
An Old Concept: Move-to-front, Splay Trees, …

- Classic data structures: lists, trees

- Linked list: move frequently accessed elements to front!



- Trees: move frequently accessed elements closer to root



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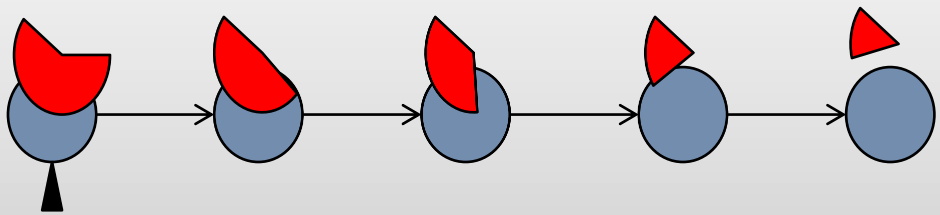

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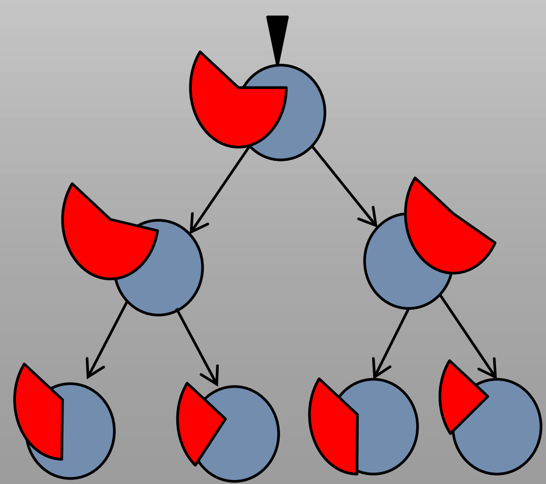

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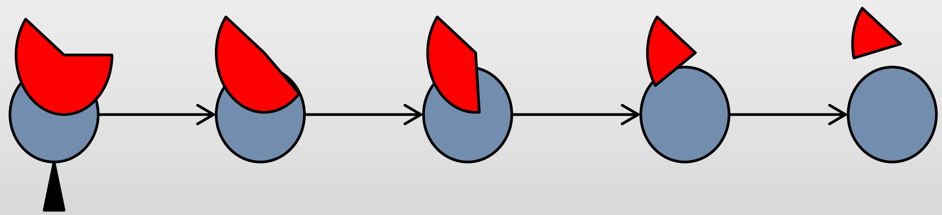

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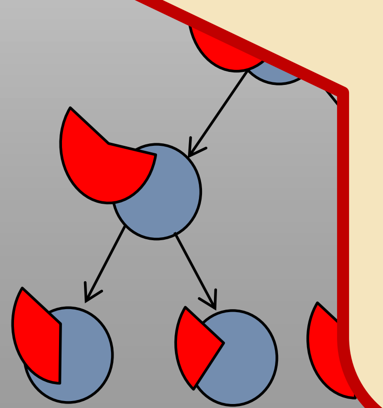
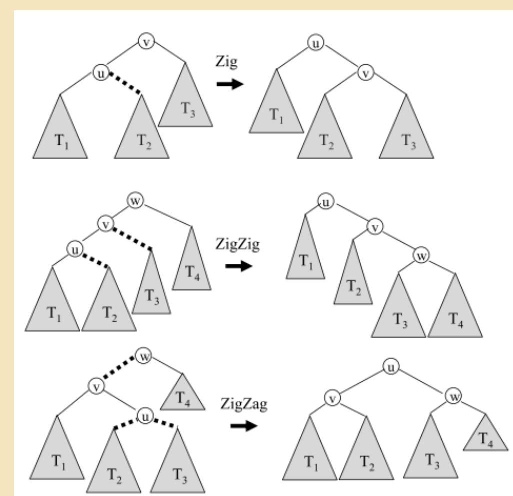


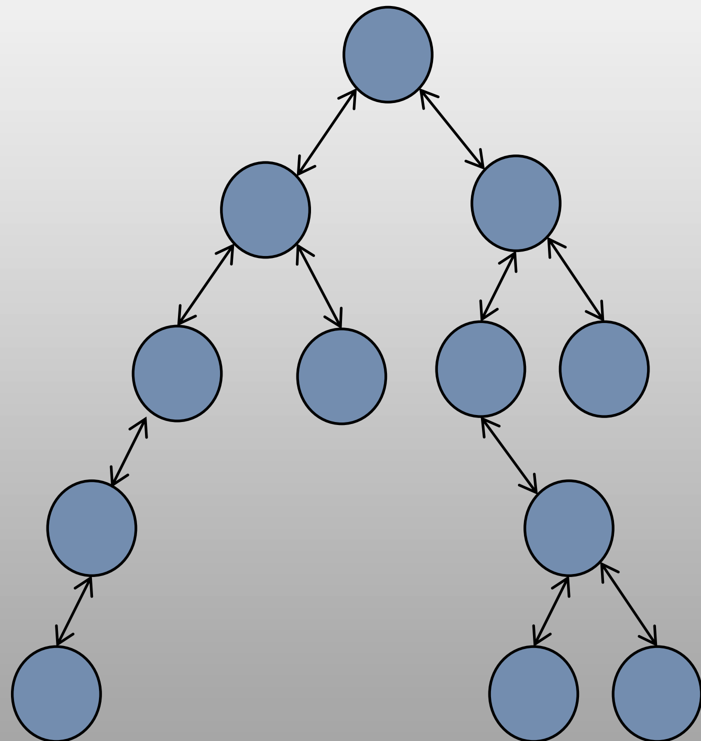
Splay Trees!



Stefan Schmid (T-Labs)

The Vision: Splay Networks ("Distributed Splay Trees")

- Most simple self-adjusting tree network: Binary Search Tree (BST)



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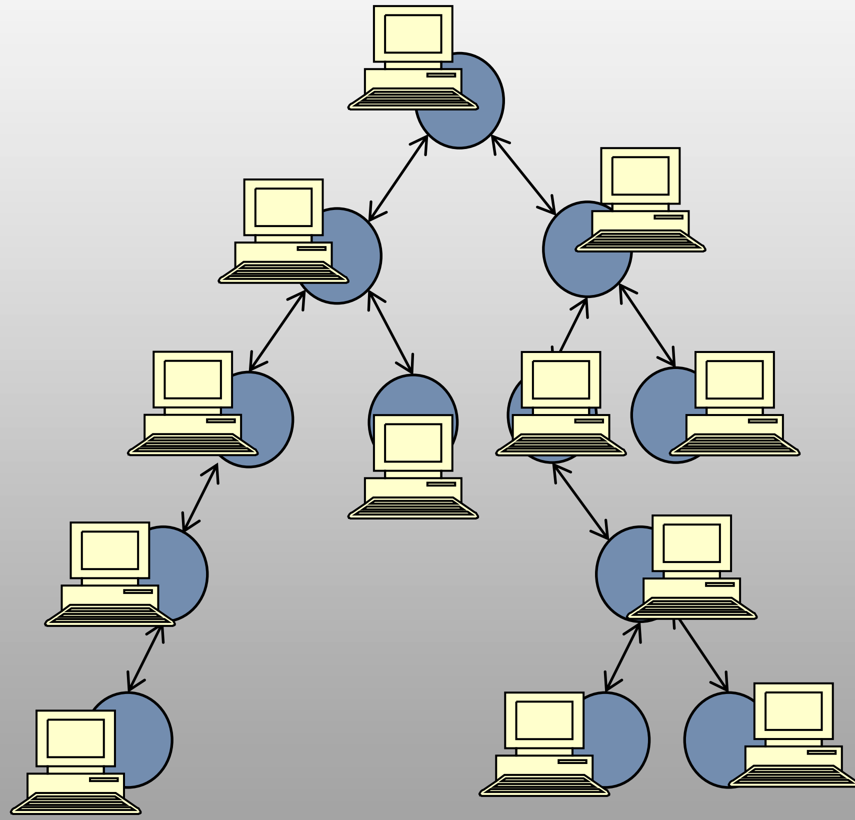

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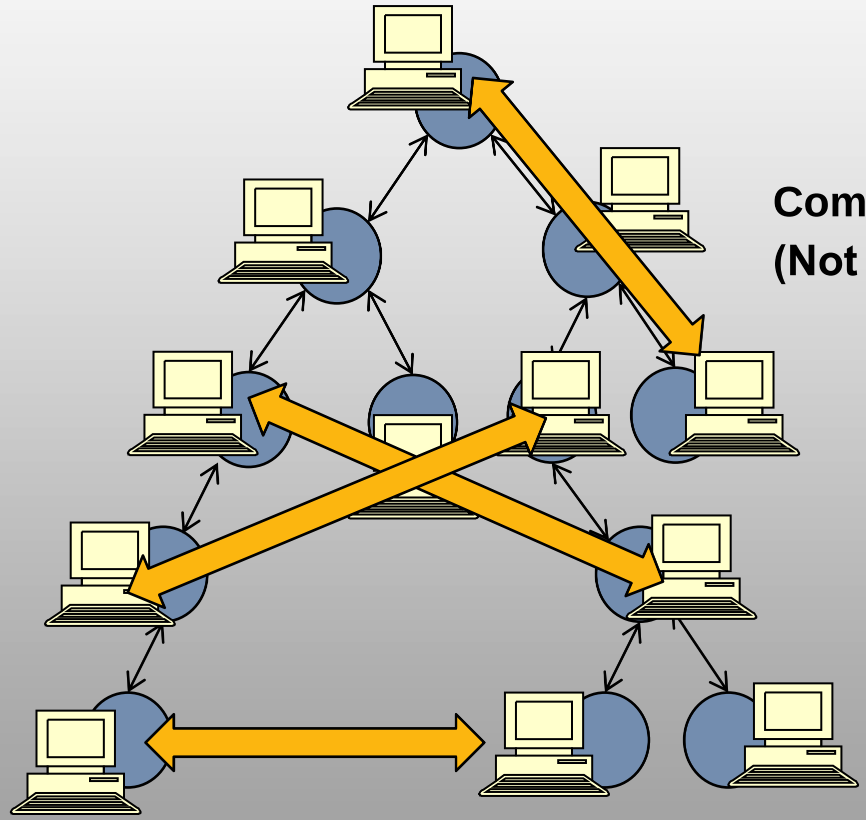
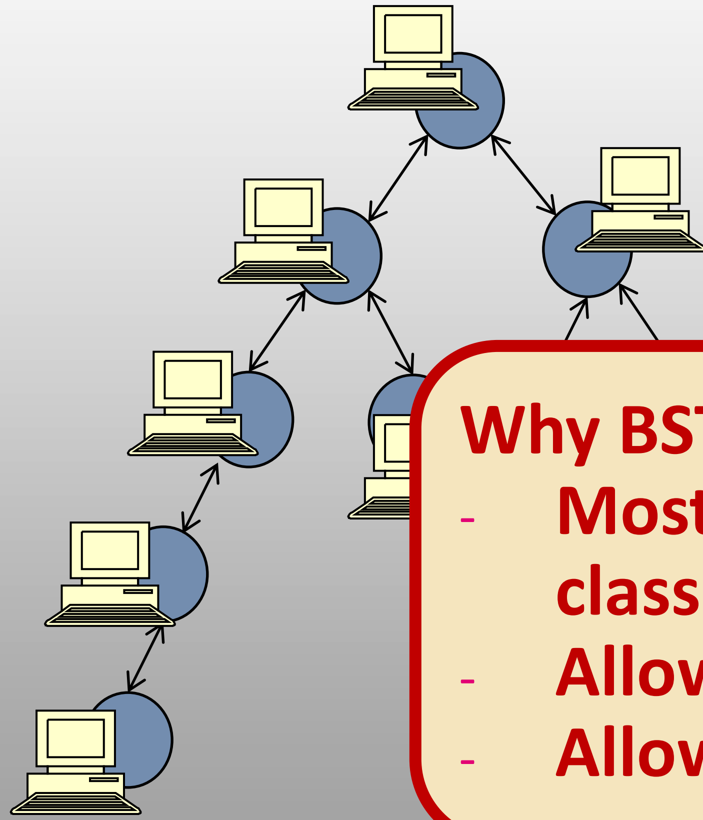


Communication between peer pairs!
(Not only lookups from root…)

The Vision: Splay Networks ("Distributed Splay Trees")

- Most simple self-adjusting tree network: Binary Search Tree (BST)



Why BST?!
- **Most simple generalization of classic data structure**
- **Allows for local routing!**
- **Allows for algebraic gossip**

Model: Self-Adjusting SplayNets

Input:

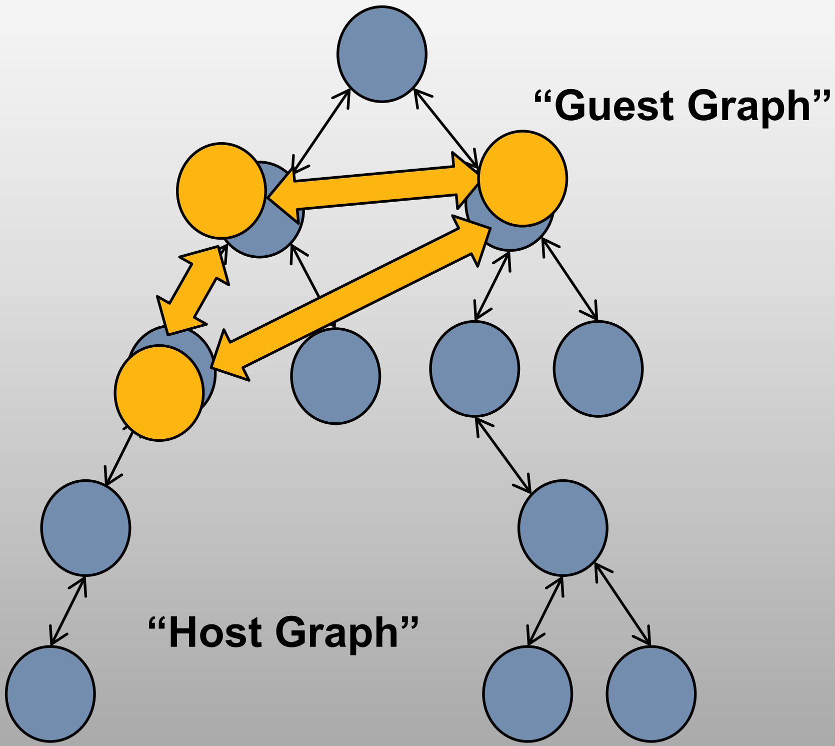
- communication pattern:
  (static or dynamic) graph

Output:

- sequence of network adjustments

Cost metric:

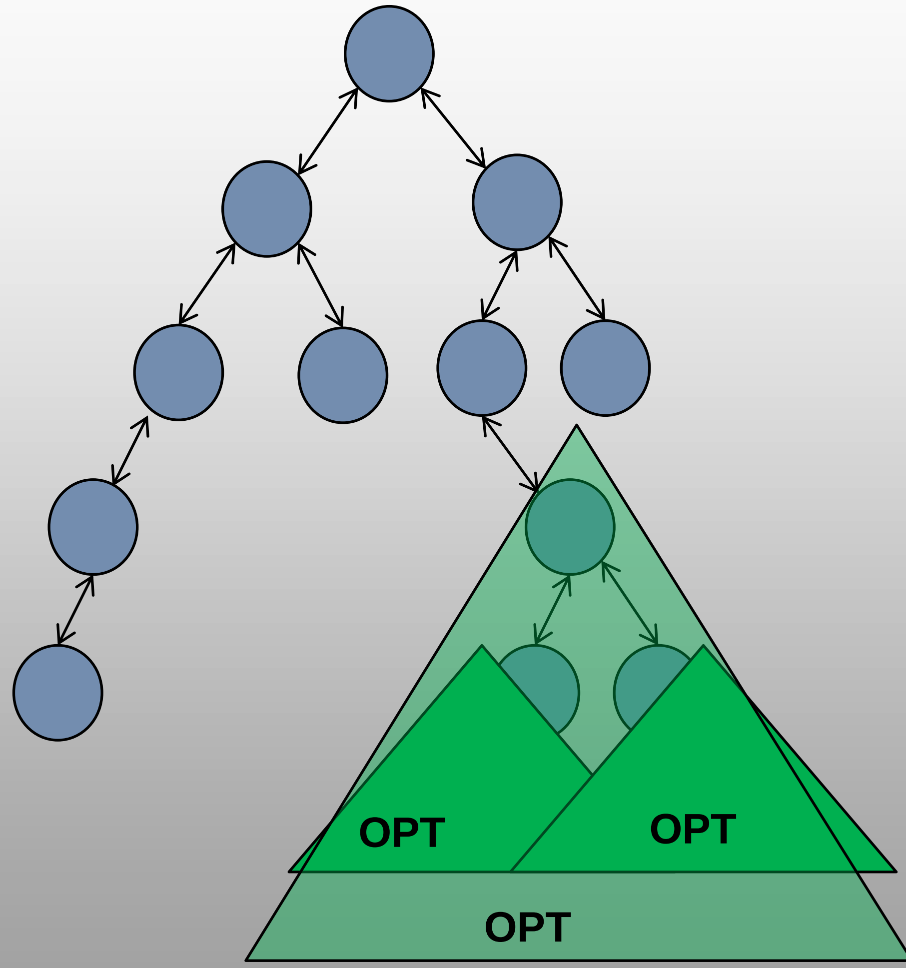
- expected path length
- # (local) network updates



The Optimal Offline Solution

Dynamic program

- Binary search:
  decouple left from right!
- Polynomial time
  (unlike MLA!)
- So: solved M"BST"A



See also:

- Related problem of
  phylogenetic trees

The Online SplayNets Algorithm

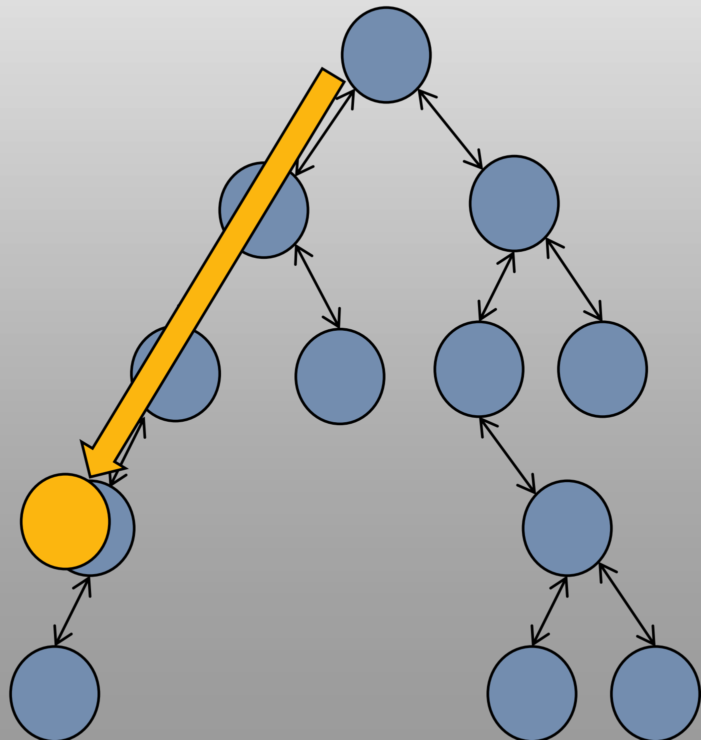
From Splay tree to SplayNet:

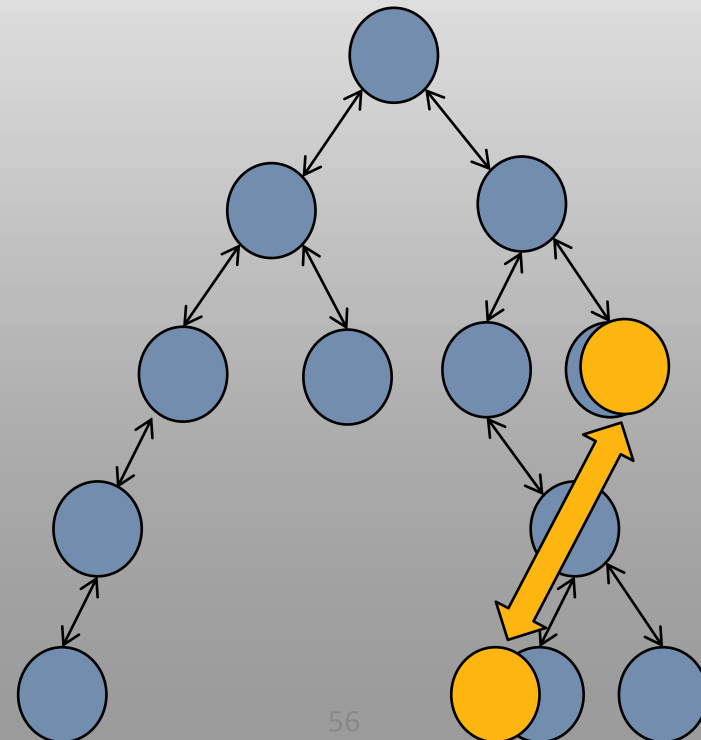
Algorithm 1 Splay Tree Algorithm $\texttt{ST}$

(* upon lookup (u) *)
splay u to root of T

Algorithm 2 Double Splay Algorithm $\texttt{DS}$

(* upon request (u, v) in T *)
$w := \alpha_T(u, v)$
$T' :=$ **splay** u to root of $T(w)$
splay v to the child of $T'(u)$



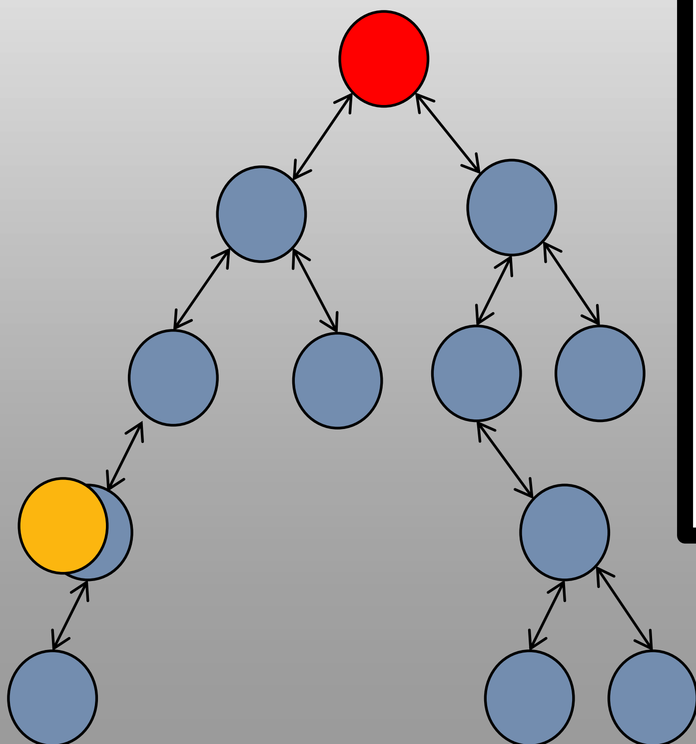


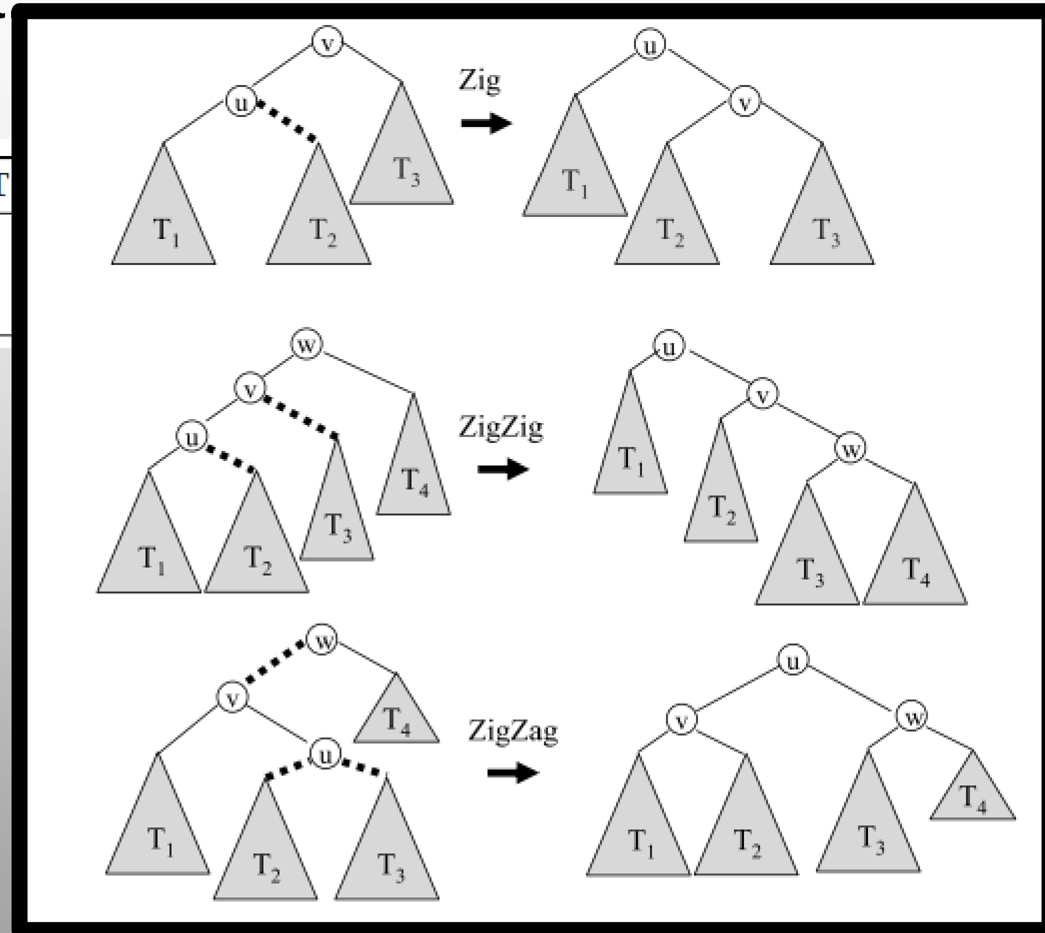
The Online SplayNets Algorithm

From Splay tree to SplayNet:

Algorithm 1 Splay Tree Algorithm ST

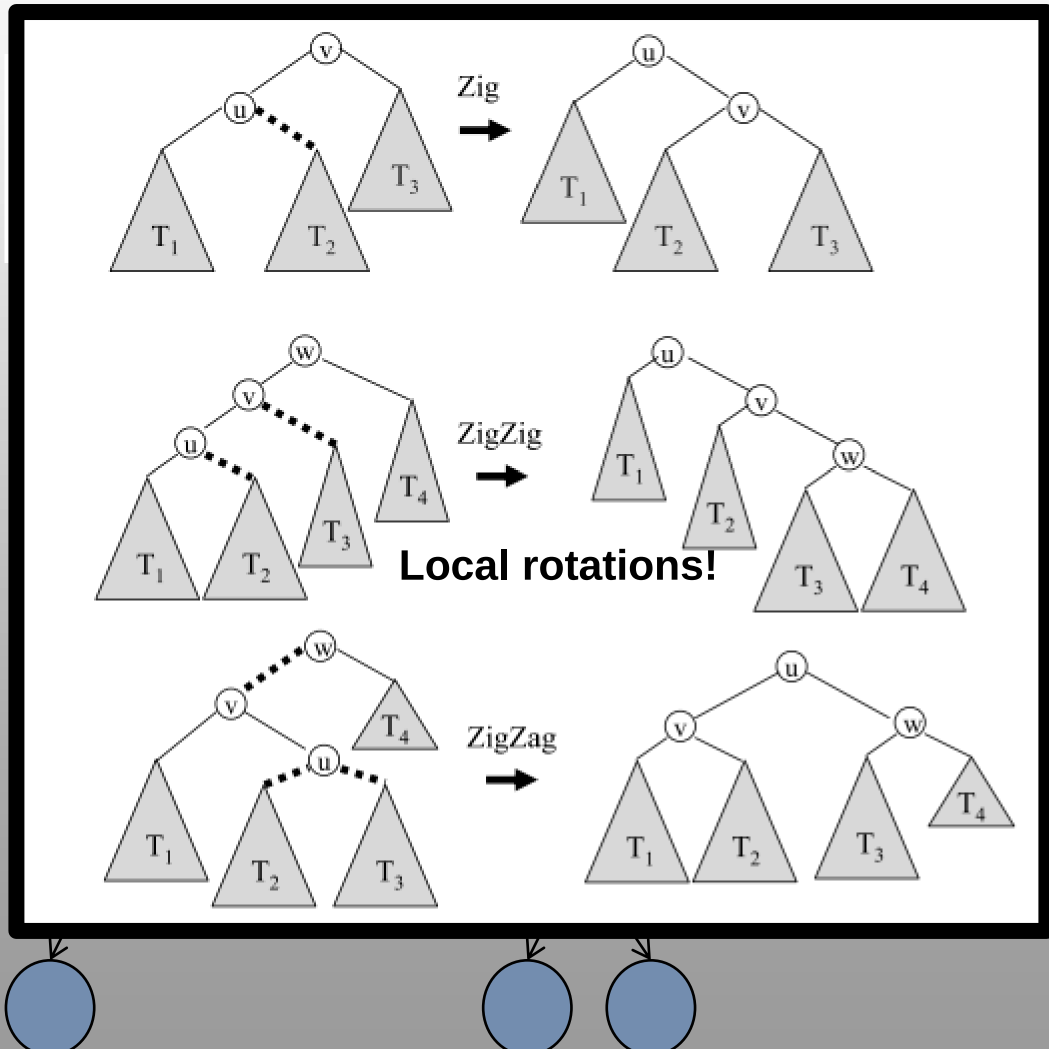
1: (* upon lookup (u) *)
2: **splay** u to root of T





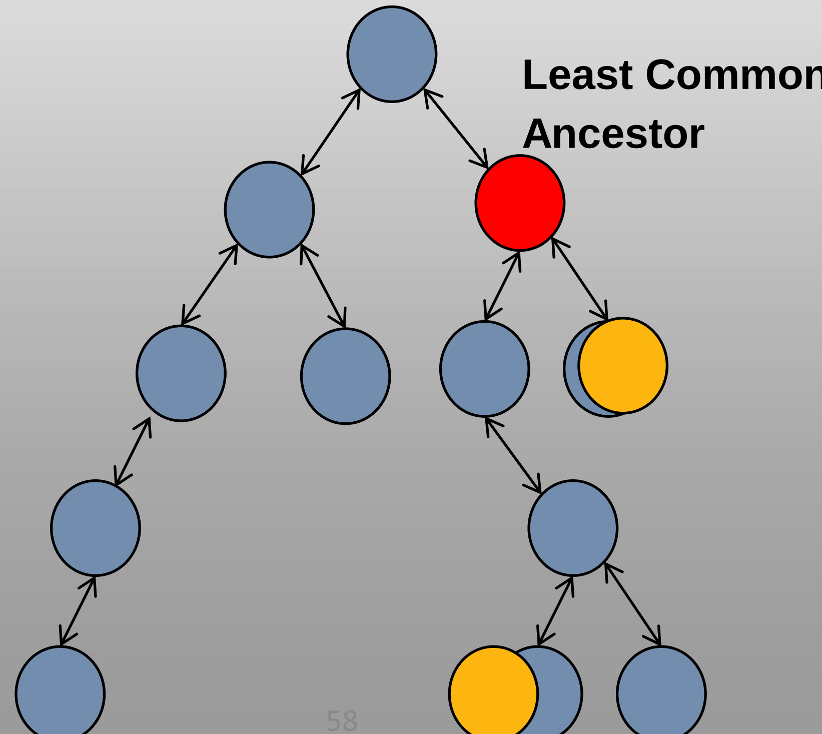
The Online SplayNets Algorithm

From Splay tree to SplayNet:



Algorithm 2 Double Splay Algorithm DS

1: (* upon request (u, v) in T *)
2: $w := \alpha_T(u, v)$
3: $T' :=$ **splay** u to root of $T(w)$
4: **splay** v to the child of $T'(u)$



Analysis: Basic Lower and Upper Bounds

Upper Bound

A-Cost < H(X) + H(Y)

where H(X) and H(Y) are empirical entropies of sources resp. destinations

Adaption of Tarjan&Sleator

Lower Bound

A-Cost > H(X|Y) + H(Y|X)

where H(|) are conditional entropies.

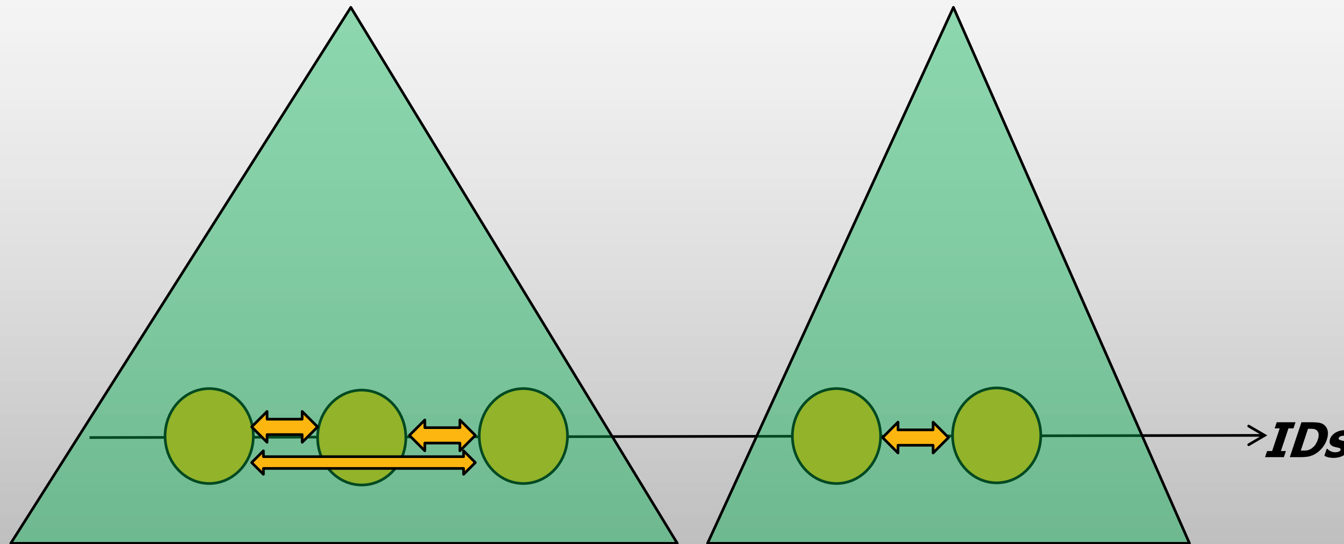
Assuming that each node is the root for "its tree"

Therefore, our algorithm is optimal, e.g., if communication pattern describes a product distribution!

Properties: Convergence

Cluster scenario:

Nodes communicate within local clusters only!

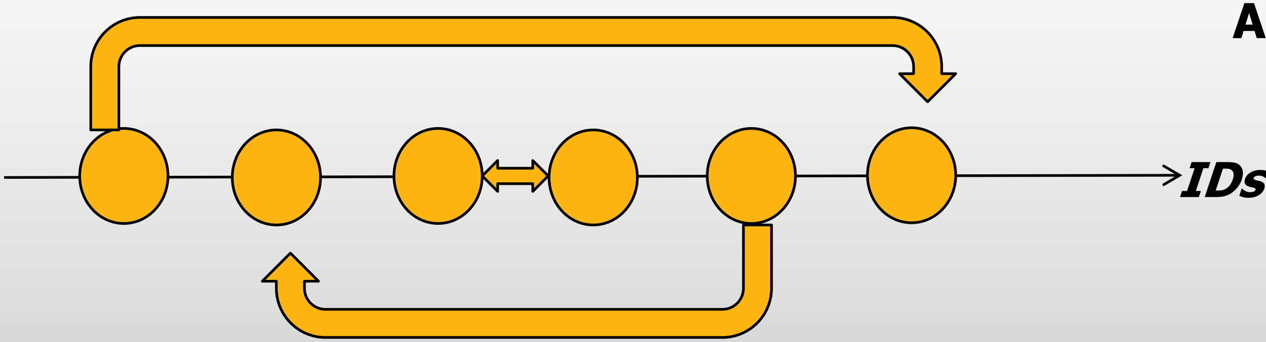


Over time, nodes will form clusters in BST! No paths "outside".

Properties: Optimal Solutions

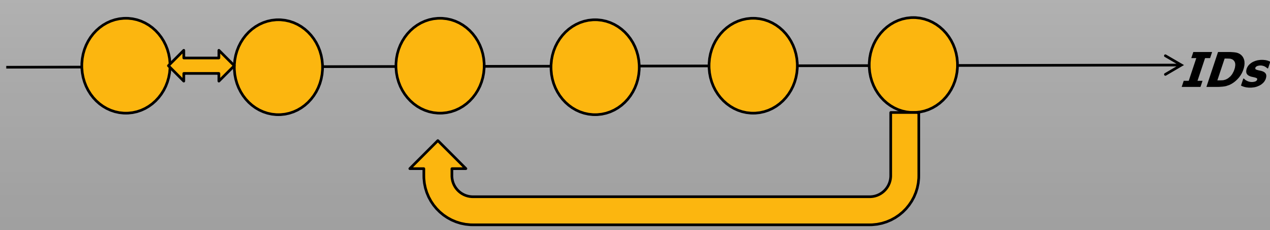
Laminated scenario:

Will converge to optimum: Amortized costs 1.



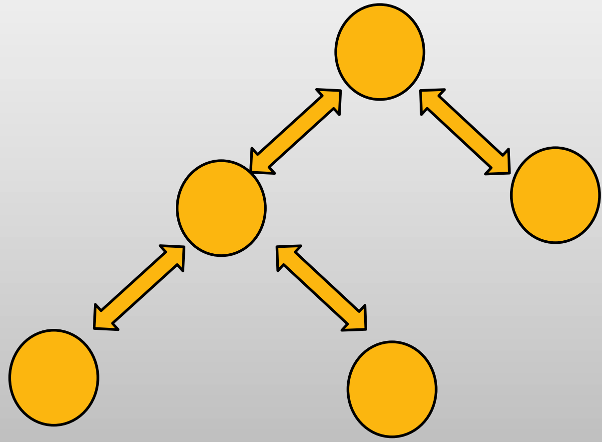
Non-crossing matching (= “no polygamy”) scenario:

Will converge to optimum: Amortized costs 1.

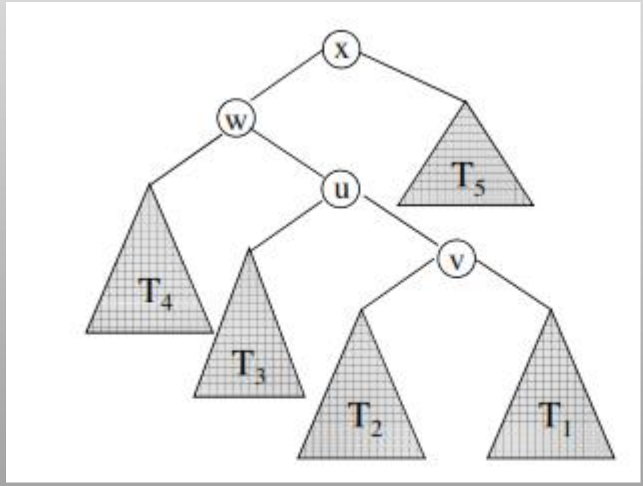


Properties: Optimal Solutions

Multicast scenario (BST): Example

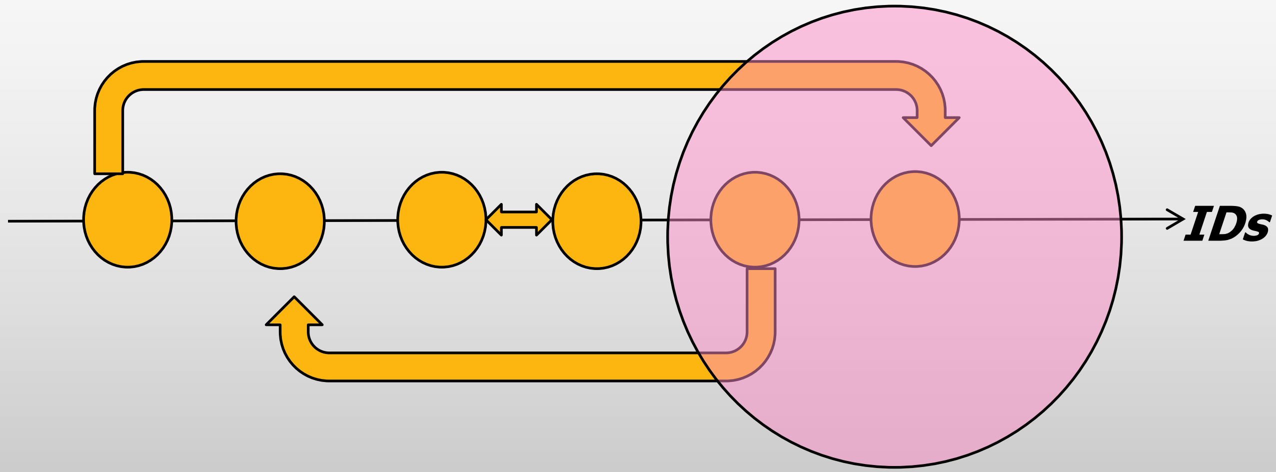


Invariant over “stable” subtrees (from right):

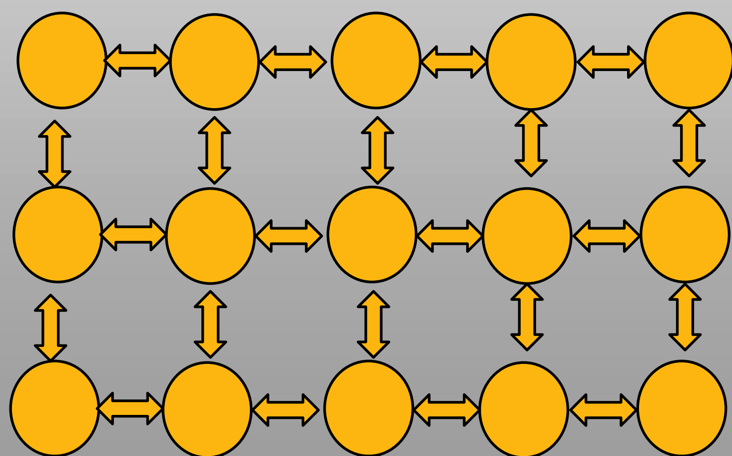


Improved Lower Bounds (and More Optimality)

Via interval cuts or conductance entropy:

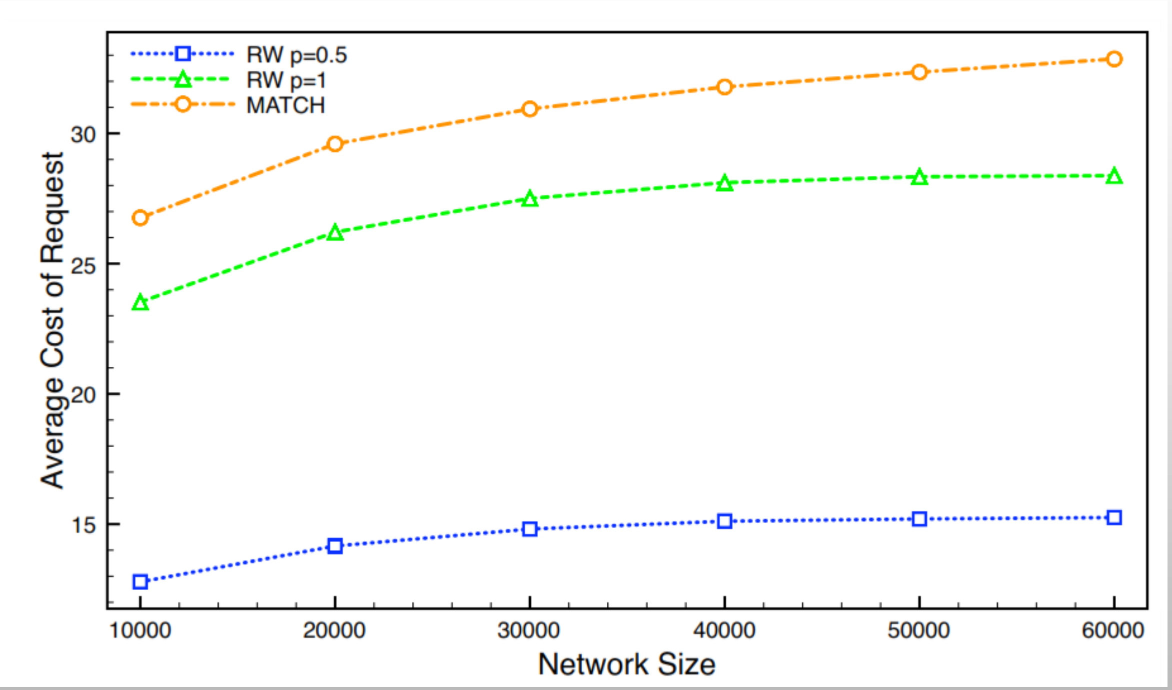


Grid:



Cut of interval: entropy yields amortized costs!

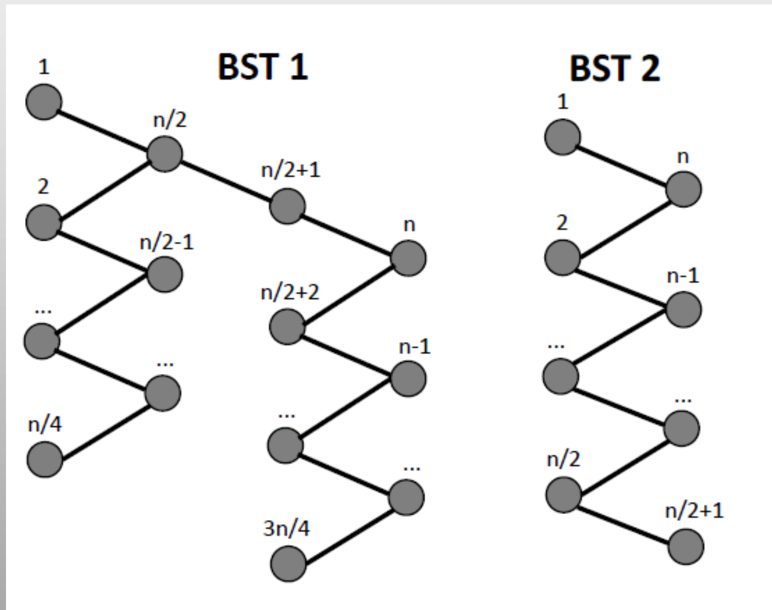
Simulation Results

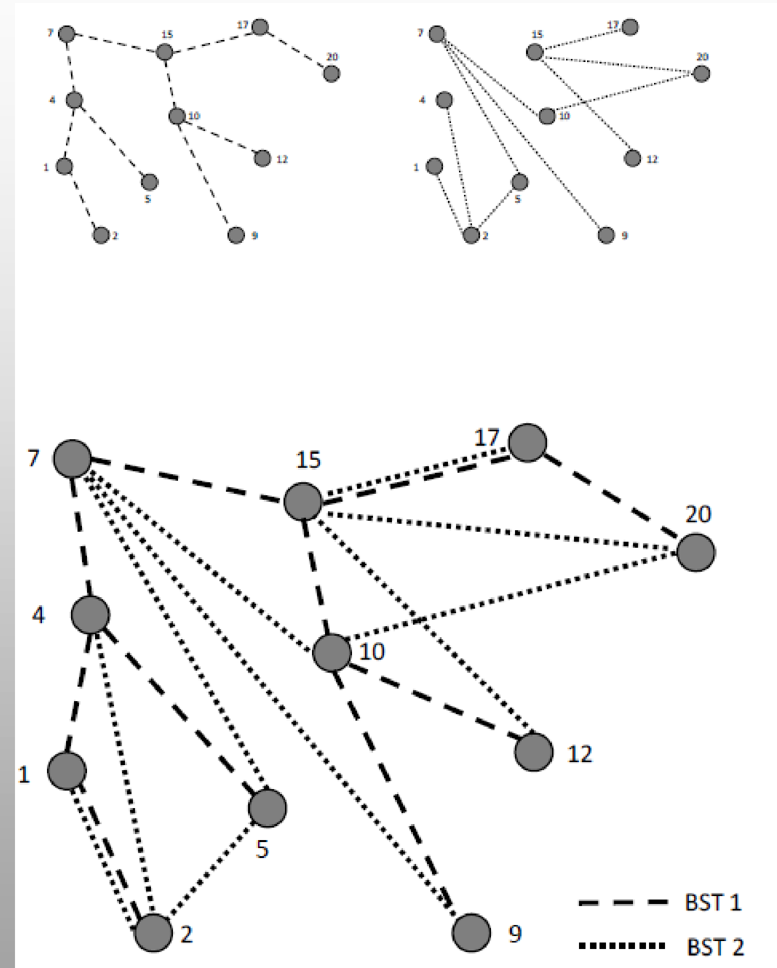


- Facebook component with 63k nodes and 800k edges
- SplayNet exploit random walk locality, to less extent also matching

Multiple BSTs: OBST

- Static:
  - Not much help for lookup model
  - Much help for routing model!





- Dynamic: yes ☺

Conclusion

- Topological self-stabilization
  - Linearization, Skip Graph, Delaunay
  - Take-home messages: local checkability, only one single phase possible, compute “local” version but ensure connectivity, …
- Self-optimization
  - Beating the lower bounds
  - First look into trees
  - Related to entropy
- Papers: PODC 2009, ISAAC 2009, LATIN 2010, IPDPS 2013, P2P 2013, etc.

Thank you! Questions?