Universität Bremen

ALTA
Institute for Algebra, Geometry, Topology and their applications

Applied and Computational Algebraic Topology
July 15-19, 2013

Speaker: Dominique Attali, CNRS Grenoble, France

Title: Collapsing Rips complexes for shape reconstruction in high dimensions

Abstract: In many practical situations, the object of study is only known through a finite set of possibly noisy sample points. It is then desirable to try to recover the geometry and the topology of the object from this information. In this talk, we will focus on an approach that approximates a shape from a set of sample points by returning the Rips complex of the points. Given a point set P and a scale parameter α, the Rips complex is the simplicial complex whose simplices are subsets of points in P with diameter at most 2α. Rips complexes have generally a size and dimension much too large to allow an explicit representation. Nonetheless, Rips complexes enjoy the property to be completely determined by the graph of theirs vertices and edges which thus provide a compressed form of storage (quadratic in the number of data points and linear in the ambient dimension). This suggests to reconstruct a shape by first building the Rips complex of the data points at some scale (encoded with its vertices and edges) and second by simplifying the result through a sequence of elementary operations [3]. We formulate conditions under which the Rips complex of the point set at some scale reflects the homotopy type of the shape [1, 4].We then give conditions under which the complex can be transformed by a sequence of collapses into a triangulation of the shape [2]. This is a joint work with André Lieutier and David Salinas.

Speaker: Paul Bendich, Duke University, USA

Title: Towards Statistics on Vineyards with Fuzzy Frechet Means

Abstract: In order to use persistence diagrams as a true statistical tool, it would be very useful to have a good notion of mean and variance for a set of diagrams. Several years ago, Mileyko and his collaborators made the first study of the properties of the Frechet mean in (D_p, W_p) the space of persistence diagrams equipped with the p-th Wasserstein metric. In particular, they showed that the Frechet mean of a finite set of diagrams always exists, but is not necessarily unique. As an unfortunate consequence, one sees that the means of a continuously-varying set of diagrams do not themselves vary continuously, which presents obvious problems when trying to extend the Frechet mean definition to the realm of vineyards. We fix this problem by altering the original definition of Frechet mean so that it now becomes a probability measure on the set of persistence diagrams; in a nutshell, the mean of a set of diagrams will be a weighted sum of atomic measures, where each atom is itself the (Frechet mean) persistence diagram of a perturbation of the input diagrams. We show that this new definition defines a (Hoelder) continuous map, for each k, from $(D_p)^k \to P(D_p)$, we discuss the ways in which it is an extensionn the original definition, and we present several examples to show how it may become a useful statistic on vineyards. This is joint work with Elizabeth Munch, Katharine Turner, John Harer, Sayan Mukherjee, and Jonathan Mattingly.

Speaker: Peter Bubenik, Duke University, USA

Title: Metrics on diagrams and persistent homology

Abstract: I will show how many flavors of persistent homology, including sublevelset, levelset, multidimensional, and angle-valued persistent homology can be understood using diagrams of ordered metric spaces. In this common framework interleavings provide a metric and functoriality implies stability. If the diagrams lie in an abelian category (e.g. persistence modules) then we have kernel, image and cokernel persistence and they too are stable. This is joint work with Jonathan A. Scott and Vin de Silva.

Speaker: Frederik Chazal, INRIA, France

Title: Optimal rates of convergence for persistence diagrams in Topological Data Analysis (joint work with M. Glisse, C. Labrure, B. Michel)

Abstract: Computational topology has recently known an important development toward data analysis, giving birth to the fieleld of topological data analysis. Topological persistence, or persistent homology, appears as a fundamental tool in this field. In this talk, we will study topological persistence in general metric spaces, with a statistical approach. We will show that the use of persistent homology can be naturally considered in general statistical frameworks and persistence diagrams can be used as statistics with interesting convergence properties.

Speaker: Justin Curry, University of Pennsylvania, USA

Title: Persistent Homology via Cellular Cosheaves

Abstract: One-dimensional persistent homology provides robust topological descriptors of data by sampling and aggregating over a parameter. Traditionally, one studies the sublevel sets of a parameter to get a sequence of vector spaces that are then depicted using "barcodes". Cellular cosheaves provide an alternative way of organizing multi-dimensional persistence that is custom-tailored to the application at hand, thereby producing a compressed representation of the data. In this talk, I will show how to recover 1d-persistence and give examples in higher dimensions to illustrate the approach. Moreover, I will reprove some recent theorems in persistence using cosheaf homology, which easily generalize to multi-D analogs.

Speaker: Herbert Edelsbrunner, IST, Austria.

Title: Sampled dynamical systems

Abstract: We consider dynamical systems that are discretely sampled both in time and in space. Let $f : X \longrightarrow$ X be a continuous self-map, U the homology group of X using field coefficients, and $\phi : U \longrightarrow U$ be the linear map induced in homology. We show how to compute the dimension of an eigenspace of phi using persistent homology of a multi-scale representation of a finite sampling of the self-map. Besides describing an algorithm, we present computational experiments, and formulate under what conditions it converges and gives stable results.

This is joint work with Grzegorz Jablonski and Marian Mrozek.

Speaker: Graham Ellis, National University of Ireland, Ireland

Title: Applied Computational Group Theory?

Abstract: Computational group theory is one of the oldest branches of computational algebra. The subject was born in 1911 with Dehn's algorithmic problems of deciding, for a group defined by a finite set of abstract generators and relations, whether the group is finite and whether a word in the generators represents the identity element. Despite the undecidability of these problems, the development of practical implementations of 'algorithms' for handling finitely presented groups began to flourish in the sixties. There is now a large computational group theory community who share implementations of group theoretic algorithms by contributing to the GAP and Magma systems for computational algebra.

This talk will investigate some potential uses of group theoretic algorithms in applied topology. The talk will start with a discussion of the fundamental group of knot and link complements as a descriptor of protein structures. It will end with some computations of Postnikov invariants of spaces and of the homology of the associated homotopy 2-types. The talk will advertize the suitability of the GAP system for computational algebra as an environment for sharing implementations of algorithms in applied topology.

Speaker: Lisbeth Fajstrup, Aalborg University, Denmark

Title: Cut-off theorems in PV-models, a geometric approach.

Abstract: In Dijkstra's PV-model, a program is given by its use of shared resources. There is a set of resources r^i each with a limited capacity k_i. A *thread* is a list of requests for access P_i (if granted access, the thread locks the resource) and release of resources V_i. When several threads run in parallel, it may create conflicts once the capacity of some of the resources is reached. The geometric model of one thread is a graph representing loops and branches of the thread. The geometric model of a parallel program is the product of the graphs of the threads. Some points are cut out - hyper rectangles, where the capacity of a resource is superseded. An execution is a directed path from the joint initial state of all threads to the joint final state of all states. Here, we consider the special case, where a thread T is run in parallel with itself n times, given the joint execution T^n. A *cut-off theorem* is a result that a property holds for all n, if and only if it holds up to a fixed n. We give two such theorems A deadlock is a state in T^n, where no thread can proceed, either because it reached its final state or because it requests access to a resource r^j which is locked by k_j other threads. The state where all threads have reached their final state is not a deadlock.

Theorem 1 Given a thread T which accesses resources $r^1, ..., r^l$ of capacity $k_1, ..., k_l$ Let T^n denote T in parallel with itself n times, then T^n is deadlock free for all n if and only if T^M is deadlock free, where $M = \Sigma_{j=1}^{l} k_j$, the sum of the capacities.

A joint program T^n is serialisable if all execution paths are directed homotopy equivalent to a serial execution, executing one thread at a time.

Theorem 2 Let T be a thread which accesses resources $r^1, ..., r^l$, each of capacity $k = 1$. Then T^n is serialisable if and only if T^2 is serialisable.

Speaker: Michael Farber, University of Warwick, United Kingdom

Title: Geometry and Topology of random 2-complexes

Abstract: In the talk I will discuss geometric and topological properties of random 2-complexes. One of the central questions is whether one can generate randomly aspherical 2-complexes (i.e. such that $\pi_2(Y) = 0$) and whether random aspherical 2-complexes satisfy the Whitehead Conjecture. This conjecture was proposed in 1941 by J.H.C. Whitehead; it states that any subcomplex of an aspherical 2-complex is also aspherical.

A result presented in the talk states that (under certain assumptions) any aspherical subcomplex $Y' \subset Y$ of a random 2-complex Y satisfies the Whitehead conjecture, with probability tending to 1. The other results describe torsion in the fundamental groups of random 2-complexes. The proofs use Cheeger constants and systoles of simplicial surfaces as well as properties of Gromov hyperbolic groups. The talk is based on a joint work with Armindo Costa.

Speaker: Patrizio Frosini, University of Bologna, Italy

Title: Adapting persistent homology to invariance groups

Abstract: It is well known that persistent homology is invariant under the action of the group of all homeomorphisms. This means that if $f : X \to R$ is a filtering function on the topological space X and $h : X \to X$ is a homeomorphism, then the persistent homology of f is equal to the one of $f \circ h$. This is a relevant obstacle to the use of persistent homology in shape comparison, since this invariance seems to be unsuitable for many applications. As an example, let us consider some filtering functions from the real plane to R, describing the grey-level images of the alphabetic letters, possibly after normalizing their global extrema. We observe that these functions are defined on the same topological space, and that the ones representing the letters “A”, “D”, “O”, “P”, “Q”, “R” are similar to each other with respect to persistent homology. Obviously, in this case a proper subgroup of the group of all self-homeomorphisms of the real plane could be preferable as a maximal invariance group of our shape descriptor.

A possible approach to this problem consists in changing the data we work on, for example extracting the boundaries of the letters in the images, and applying persistent homology to some new filtering function defined on these boundaries. However, this process has a computational cost, and leads to manage topological spaces that can have different homotopy types, as happens when we compute the boundaries corresponding to the images of a letter “A” and a letter “B”. Furthermore, it is not clear which filtering functions we should define on the boundaries in order to obtain the invariance we need. Other analogous approaches present similar problems.

In this talk we illustrate a different technique to adapt persistent homology to invariance groups that are contained in the group of all self-homeomorphisms of the topological space X. It is based on two methods that are related to the choice of suitable functionals, acting on the set of all possible filtering functions.

Speaker: Eric Goubault, CEA Saclay, France

Title: Determination of trace spaces, and the geometric nature of synchronization

Abstract: This talk will mostly consist of an assembly of some known, but scattered, results, both in computer science and in topology, and some of their (simple) consequences. A striking fact is the existence of relatively efficient methods for analyzing concurrent models with mutual exclusion (binary semaphores only), based on (safe) Petri nets or (prime) event structures, whereas things seem to be much more intricate in classical approaches to concurrency, when it comes to dealing with more subtle synchronization primitives, such as, for instance, counting semaphores. In the directed topological approach to these concurrent models, this difference is apparent since trace spaces in the mutual exclusion onlymodel appear to be very simple. In fact even, in the latter case, the state space is geometrically a non-positive curvature (NPC) space (or cubical complex, in the discrete setting). As observed by Ghrist et al., the L^∞ geodesics are then discrete up to homotopy (for finite NPC cubical complexes), and in fact they correspond to traces, up to dihomotopy.

Also, as a direct consequence of work by Ardila et al., NPC cubical complexes are the same as prime event structures introduced to model, precisely, concurrent processes with mutual exclusion some 25 years before, and equivalent to safe Petri nets, introduced and heavily studied even before. We explore some of the consequences of these facts in the rest of the talk, among which, the link to trace space computation algorithms (Raussen et al.) and generalizations, homological computations of the dihomotopy classes of dipaths, and extensions, if time permits, to persistent homological characterizations of classes of dipaths.

Speaker: Rick Jardine, University of Western Ontario, Canada

Title: Homotopy theories of dynamical systems

Abstract: A dynamical system is, generally, an action $X \times S \to X$ of a parameter space S on a space X. This talk will explore what is meant by the assertion that one dynamical system is homotopically equivalent or closeto another. There are various possibilities, which involve naively varying just the S-space X, or the spaces X and S together. The latter is more interesting.

Speaker: Thomas Kahl, University of Minho, Braga, Portugal

Title: On topological abstraction of higher dimensional automata

Abstract: One of the most expressive models of concurrency is the one of higher dimensional automata. A higher dimensional automaton (HDA) is a labelled precubical set. In this talk, I will discuss topological abstraction of HDAs, i.e. the replacement of HDAs by more abstract and smaller ones that can be considered equivalent from a topological point of view. I will introduce two preorder relations for HDAs, called trace equivalent abstraction and homeomorphic abstraction. These preorder relations are defined using certain label preserving d-maps called weak morphisms. Roughly speaking, a weak morphism between two HDAs is a continuous map between their geometric realisations that sends subdivided cubes to subdivided cubes and that preserves labels of paths. An HDA A is said to be a trace equivalent abstraction of an HDA B if there exists a weak morphism from A to B that preserves the trace category, which is a variant of Bubenik's fundamental bipartite graph. In order to define homeomorphic abstraction, one proceeds similarly using weak morphisms that are homeomorphisms. Homeomorphic abstraction may be seen as a labelled version of T-homotopy equivalence in the sense of Gaucher and Goubault. The main result on homeomorphic and trace equivalent abstraction is that the former is essentially always stronger than the latter.

It is also possible to define the trace language of an HDA and to show that, for a large class of HDAs, it is invariant under trace equivalent abstraction.

Speaker: Matthew Kahle, Ohio State University, USA

Title: Topology of random flag complexes

Abstract: We will discuss a recent cohomology vanishingtheorem for random flag complexes which provides a generalization of the Erdos-Renyi theorem to higher dimensions. The method goes back to seminal work of Garland, which allows one to prove cohomology-vanishing by establishing large enough spectral gaps of certain Laplacian operators. We also require new results for spectral gaps of Erdos-Renyi random graphs, which was done in recent work with Hoffman and Paquette.

Speaker: Kevin Knudson, University of Florida, USA

Title: Syzygies and multi-dimensional persistence

Abstract: Studies of multi-dimensional persistent homology typically focus on graded modules of homology groups, but there are insights to be gleaned from backing up a step and considering the graded modules of chains. In this talk I will present several examples and discuss the connection between the syzygies of these modules and persistence.

Speaker: Sanjeevi Krishnan, University of Pennsylvania, USA

Title: Higher dimensional flow-cut dualities

Abstract: Some optimization dualities, such as the max-flow min-cut theorem, are trivial cases of a Poincare Duality for (co)homology on sheaves of semimodules. This talk will present the theorem, give examples of other flow-cut dualities (e.g. smooth, higher-dimensional, monoid-theoretic) that arise from the theorem, and work out some relevant and illuminating calculations. No familiarity with semimodule theory will be assumed.

Speaker: Vitaliy Kurlin, Durham University, United Kingdom

Title: A persistence-based reconstruction of homotopy types of graphs from noisy samples in the plane

Abstract: Let a point cloud C be a noisy dotted image of a graph G in the plane. The Vietoris-Rips complex $VR(d)$ of C is defined by forming a simplex for every finite set of points of C that has diameter at most d. Persistent homology groups in dimensions 0 and 1 are computed to automatically identify a long enough interval of the parameter d when $VR(d)$ has stable components and a stable homotopy type. It is proved that the algorithm correctly reconstructs the homotopy type of G without any user-defined parameters under explicit restrictions on an unknown graph G and its given noisy sample C in the plane.

Speaker: Sefi Ladkani, University of Bonn, Germany

Title: Derived categories arising from combinatorial data

Abstract: Triangulated categories in general, and derived categories in particular, have found applications in diverse areas of mathematics and mathematical physics, forming bridges between algebra and geometry. We propose to investigate derived categories arising from combinatorial objects. Examples include partially ordered sets (posets), triangulations of surfaces and certain quivers with relations. Our main goal is to understand how the combinatorial properties of these objects are reflected in the representation theoretic and homological properties of the associated derived categories. One of the main questions concerns the existence of an algorithm that given two such objects decides whether their derived categories are equivalent, or not. In many interesting cases these questions can be answered affirmatively. We will demonstrate this by presenting several results concerning derived equivalence classifications.

Speaker: Claudia Landi, University of Modena, Italy

Title: A bridge between continuous and discrete multidimensional persistent homologies

Abstract: The theory of multidimensional persistent homology was initially developed for simplicial complexes ?ltered through an ordering of the simplices. On the other hand, stability of rank invariants is proved for topological spaces filtered by continuous vector-valued functions. This talk aims to provide a bridge between the continuous setting, where stability properties hold, and the discrete setting, where actual computations are carried out. The existence of this bridge is not obvious due to the phenomenon of a structural gap between the two settings which appears in the multidimensional case when using the standard piecewise linear interpolation of the discrete model. We solve the problem by introducing an adapted axis-wise linear interpolation and develop a stability preserving method for comparing rank invariants of vector functions obtained from discrete data. As a practical implication of our theoretical results, we present a procedure to predetermine to which extent data resolution can be coarsened while maintaining a certain error threshold on rank invariants. This talk is based on a joint work with N. Cavazza, M. Ethier, P. Frosini, and T. Kaczynski.

Speaker: Facundo Memoli, University of Adelaide, Australia

Title: Curvature sets over persistence diagrams

Abstract: A combinatorial idea of Gromov is to assign to each metric space the collection $\big\{K_n(X)\big\}_{n\in\mathbb{N}}$ of sets each consisting of all distance matrices corresponding to all possible n-tuples of points in X. It is known that $K_n(\cdot)$ is 2-Lipschitz w.r.t. the Gromov-Hausdorff distance.

The proposed extension is: given a filtration the functor $\mathcal{F}$ on finite metric spaces consider $K_n^{\mathcal{F}}(X)$, the *set of all possible $\mathcal{F}$-persistence diagrams generated by metric subsets of X of cardinality n*. Is there a sense in which $K_n^{\mathcal{F}}$ is stable? For a class of filtration functors which we call *compatible*, the answer is positive, and these admit stability results in the Gromov-Hausdorff sense.

In order to capture frequency or statistics, it is more useful to consider that, in addition to a metric structure, a probability measure has been specified. Then, to an mm-space X one assigns the collection $\{U_n(X)\}_{n\in\mathbb{N}}$ of all measures each defined on $\mathbb{R}^n \times \mathbb{R}^n$ each and given by the pushforward of the n-fold product measure $\mu_X^{\otimes n}$ under the map
$\Phi_n : X^{\times n} \to \mathbb{R}^n \times \mathbb{R}^n$ which sends $(x_1, x_2, \ldots, x_n)$ to the distance matrix $\big(d_X(x_i, x_j)\big)_{i,j=1}^n$. The link is that $\mathrm{supp}\big(U_n(X)\big) = K_n(X)$.

This construction can be adapted to give a *measured version of* $K_n^{\mathcal{F}}$ which can encode the statistics of persistence diagrams arising according to a given filtration functor $\mathcal{F}$. The proposal is to consider the pushforward measure $U_n^{\mathcal{F}}(X) := (D_* \circ \mathcal{F} \circ \Phi_n)_{\#}\, \mu_X^{\otimes n}$ induced on barcode space. Tthis measured version is obviously connected with the combinatorial construction: $\mathrm{supp}\big(U_n^{\mathcal{F}}(X)\big) = K_n(X)$. The stability of these constructions can now be expressed in Gromov-Wasserstein sense.

Speaker: Roy Meshulam, Technion, Haifa, Israel

Title: Topology and Combinatorics of Ramanujan Complexes

Abstract: Ramanujan complexes are a certain family of finite quotients of affine buildings associated with the linear group over a local field. Ramanujan complexes are on one hand highly structured, e.g. all their vertex links are isomorphic to the order complex of subspaces of a finite vector space.

On the other hand they exhibit random like properties which makes them (potentially) useful in various extremal problems in topological combinatorics. We'll discuss some applications of Ramanujan complexes, including e.g. their essential optimality with respect to the higher dimensional Moore bound for the diameter of a top dimensional cycle in a simplicial complex. Joint work with Alex Lubotzky.

Speaker: Dmitriy Morozov, LBNL, USA

Title: Back to Basics: Merge Trees

Abstract: This talk revisits merge trees, a basic topological descriptor that records connectivity of sublevel sets of a scalar function. We introduce an interleaving distance between two merge trees and establish its stability to perturbations of the function. We show that this distance is never smaller than the bottleneck distance between 0-dimensional persistence diagrams of the function. On the computational side, we consider a distributed representation of merge trees that not only improves their parallel computation, but also supports parallel analysis. As an example, we show how to extract a prescribed levelset component of the function with minimum communication.

Speaker: Neza Mramor Kosta, Univerity of Ljubljana, Slovenia

Title: Birth and death in discrete Morse theory

Abstract: Suppose M is a finite cell complex and that for finitely many values $0 = t_0 < t_1 < \cdots < t_r = 1$ of t we have a discrete Morse function $F_{t_i} : M \to \mathbb{R}$. We will discuss the births and deaths of critical cells for the functions F_{t_i} as t increases and present an algorithm for pairing the cells that occur in adjacent slices, first in the case where the cell decomposition of M is the same for

each t_i, and then in the case where they may differ. The algorithm is correct in the sense that given a smooth manifold M and a smooth 1-parametric family of functions f_t on M that are generically Morse, there exists triangulations of $M_1, \ldots M_r$ of M and discrete Morse functions F_{t_i} on M_i such that the algorithm connects all critical cells that correspond to critical points connected in the bifurcation diagram of f. This has potential applications in topological data analysis, where one has function values at a sample of points in some region in space at several different times or at different levels in an object.

Speaker: Daniel Müllner, Stanford University, USA

Title: Stability of levelset zigzag persistence and discretized Reeb graphs.
Joint work with Aravindakshan Babu and Gunnar Carlsson.

Abstract: We set up a framework to study the behavior of levelset zigzag persistence (Carlsson, de Silva, Morozov, 2009) under coarsening and refinement of covering intervals. This also covers the situation where the exact location of critical values is not known. We use this approach to give approximation guarantees on the levelset zigzag persistence of discretized Reeb graphs obtained from point clouds, as they are produced by the one-dimensional Mapper algorithm (Singh, Mémoli, Carlsson, 2007).

Speaker: Martin Raussen, Aalborg University, Denmark

Title: Spaces of directed paths as simplicial complexes

Abstract: Concurrency theory in Computer Science studies the effects that arise when several processors run simultaneously sharing common resources. It attempts to advise methods to deal with the "state space explosion problem", sometimes using models with a combinatorial/topological flavor. It is a common feature of these models that an execution corresponds to a directed path (d-path), and that d-homotopies (preserving the directions) have equivalent computations as a result.

I will discuss particular classical examples of directed spaces, a class of Higher Dimensional Automata (HDA). For such a space, I will describe a (nerve lemma) method that determines the homotopy type of the space of traces (executions) as a prodsimplicial complex – with products of simplices as building blocks. A description of that complex opens up for (machine) calculations of homology groups and other topological invariants of the trace space. The determination of path components is particularly important for applications.

Unfortunately, the resulting prodsimplicial complexes grow still quickly in both dimension and the number of cells. I shall sketch ongoing work with K. Ziemiański (Warsaw) that tries to overcome this drawback by finding smaller homotopy equivalent simplicial complexes via suitable homotopy decompositions of path spaces.

Speaker: Vin de Silva, Pomona College, USA

Title: Persistent cohomology and the topological analysis of recurrent signals

Abstract: I will present a protocol for studying the recurrence properties of time-series data, by constructing auxiliary coordinates on the signal that reveal its topological properties. These coordinates take values in the circle, rather than the real line. For instance, we discover the period of a periodic signal without any kind of Fourier analysis. The method makes it easy to discover, heuristically, the quasiperiodic behaviour of chaotic systems such as the one containing the Lorenz attractor. The main tools are Takens delay embedding, persistent cohomology, and discrete Hodge theory. This is joint work with Primoz Skraba and Mikael Vejdemo-Johansson, with contributions by Dmitriy Morozov and Konstantin Mischaikow.

Speaker: Lucile Vandembroucq, University of Minho, Braga, Portugal

Title: On Topological Complexity and related invariants

Abstract: I will discuss the relationships between Farber's Topological Complexity and related invariants such as Iwase-Sakai's Monoidal Topological Complexity, Doeraene-El Haouari's relative category and the LS-category of the cofibre of the diagonal map. In particular, I will present some new results obtained in collaboration with Jos Calcines and Jos Carrasquel which are related to the Iwase-Sakai conjecture (asserting that Topological Complexity coincides with Monoidal Topological Complexity) and the Doeraene- El Haouari conjecture (asserting that the relative category of a map f coincides with the sectional category when f admits a homotopy retraction).

Speaker: Hubert Wagner, Jagiellonian University, Poland

Title: Persistent homology in text mining

Abstract: The main topic is at the intersection of computational topology and text-mining. More precisely: we use persistent homology to analyze the structure of similarities within a corpus of text documents. First, some basic concepts from the field of text mining will be presented. With these tools we map text data into a high-dimensional space, which can be treated with topological methods. Then, we give an interpretation of the information captured by persistence. Finally, we overview the computational difficulties and survey some new results, which make the computations feasible even for large datasets.