Tanabe, Sumumu On monodromy representation of period integrals associated to an algebraic curve with bi-degree (2,2)

As one of model cases where Kontsevich's homological mirror symmetry and related conjectures (e.g. Dubrovin's conjecture, Gamma conjecture by Galkin-Golyshev-Iritani) seem to be hopeful, we point out the case of a generic \mathcal{Y} with bi-degree (n,m) in a product of projective spaces $\mathbb{P}^{n-1} \times \mathbb{P}^{m-1}$. The discriminantal loci of this family admit a covering that turns out to be a (complex) hyperplane arrangement with simple cyclic group actions. Therefore, to our mind, this family of varieties presents certain interests for those who try to get an systematic understanding of the fundamental group of the complement to complex hyperplane arrangement.

In this talk, we restrict ourselves to the case of bi-degree curve (2,2) in a product of projective lines $\mathbb{P}^1 \times \mathbb{P}^1$. We calculate two differenent monodromy representations of period integrals for the affine variety $\mathcal{X}^{(2,2)}$ obtained by the dual polyhedron mirror variety construction from \mathcal{Y} . The first method that gives a full representation of the fundamental group of the complement to singular loci (the union of a parabola and coordinate axes) relies on the generalised Picard-Lefschetz theorem. The second method uses the analytic continuation of the Mellin-Barnes integrals that gives us a proper subgroup of the monodromy group. It turns out both representations admit a Hermitian quadratic invariant form that is given by a Gram matrix of a split generator of the derived category of coherent sheaves on on \mathcal{Y} with respect to the Euler form.